Subsidizing a New Technology: An Impulse Stackelberg Game Approach

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Link to paper

(Joint work with Georges Zaccour)

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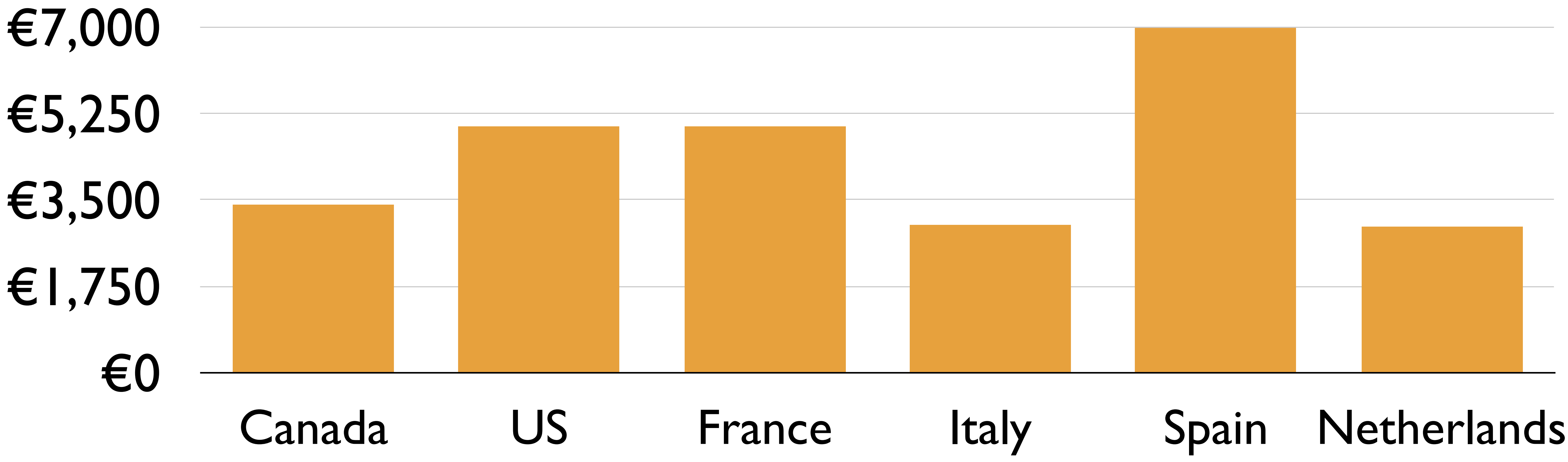
Université de Montréal GERAD CIRRELT IVADO

Economic policies need to be analyzed in terms of the incentives they create, rather than the hopes that inspired them

-Thomas Sowell

Price subsidies on electric vehicles



Why subsidize new technology?

Price subsidy

- Making firms that sell electric vehicle price competitive

    - Learning-by-doing: Unit production cost decreases with **experience** [Levitt et al., 2013]

- **Increase adoption** of electric vehicles

Levitt, S., List, J., and Syverson, C. (2013). Towards an Understanding of Learning by Doing: Evidence from an Automobile Assembly Plant. Journal of Political Economy. 121(4):643–681.

Literature: Subsidy and incentives

- **Do manufacturers fully pass over the subsidy to consumers?**

- Kaul et al (2016) analyzed the car scrappage program
  - Subsidized buyers paid little more than those who were ineligible for subsidy

- Jimenez et al. (2016): increase of €600 in car prices on average after a scrappage program was announced in Spain

Literature: new durable product diffusion

- Initiated by **Bass (1969)**
  - In 2004, voted one of the ten most influential papers published in Management Science during the last fifty years
  - **Forecasting**: through word-of-mouth communication, early adopters influence not yet adopters' purchasing decision
  - Firm is passive, **no pricing decisions**

Literature: Pricing

- Extensions of Bass Model:
  - Robinson & Lakhani (1975): Continuous-time **optimal-control** problem to determine prices
  - Eliashberg & Jeuland (1986): Two-stage model of pricing (monopoly followed by **duopoly**)
  - Dockner & Jorgensen (1988): **Price competition** in a dynamic oligopoly

Literature: Dynamic Games

- Kalish & Lilien (1983): Study the effect of **price subsidy on adoption rate**
  - Government maximizes units sold by subsidy program's terminal date
  - Lilien (1984): Application to US Photovoltaic Program
- Zaccour (1996) computed **open-loop Nash equilibrium** between government (that decides subsidy rate) and firm
- Dockner (1996) solved **Stackelberg game** with government as leader

Critique of earlier models

- Criticisms by Janssens & Zaccour (2014):
  - Different planning horizons for government and firm
  - Assumption of linear decrease in unit cost
  - Maximizing units sold is costly and inefficient
- Assumption: Subsidy can be changed at each time instant.

Our Approach

- **Our approach** (Sadana and Zaccour, 2024):
  - Government makes discrete subsidy adjustments
    - Discrete subsidy values are more realistic
  - Firm **continuously adjusts price** while government acts at **discrete time instants.**
  - We keep the assumption of unit cost linearly decreasing in cumulative sales

Literature:
Differential games with impulse control

- Nash equilibrium in nonzero-sum differential games:
  - Only impulses (no continuous controls)[1]
  - One player using impulse control and another using continuous control[2, 3, 4]

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[1] Aïd, R., Basei, M., Callegaro, G., Campi, L., and Vargiolu, T. (2020). "*Nonzero-Sum Stochastic Differential Games with Impulse Controls: A Verification Theorem with Applications.*" MOR, 45(1):205-232.

[2] Sadana, U., Reddy, P.V., Başar, T., and Zaccour, G. (2021). "*Sampled-Data Nash Equilibria in Differential Games with Impulse Controls.*" JOTA, 190(3):999-1022.

[3] Sadana, U., Reddy, P.V., and Zaccour, G. (2021). "*Nash equilibria in nonzero-sum differential games with impulse control.*" EJOR, 295(2):792-805.

[4] Sadana, U., Reddy, P.V., and Zaccour, G. (2023). "*Feedback Nash Equilibria in Differential Games With Impulse Control.*" TAC, 68(8):4523-4538.

Stackelberg game model for subsidy rollout

Target sales

- Canadian Zero-Emission Vehicles program target is 100% new light-weight vehicles sales by 2035, and it will run until March 31, 2025, or until available funding is exhausted.
- President Obama in 2011 set the target of "one million electric vehicles on the road by 2015."

Model: Two-Player Stackelberg game

- $p(t)$: electric vehicle price,
- subsidy at time t: $s(t) \in S = \{0, s_1, \cdots, s_M\}$
- $x(t)$: cumulative sales

- Sales rate:

$$\dot{x}(t) = \begin{cases} \alpha_1 + \alpha_2 x(t) - \beta(p(t) - p_a), & \text{no subsidy} \\ \alpha_1 + \alpha_2 x(t) - \beta(p(t) - s(t) - p_a), & \text{subsidy} \end{cases}$$

Demand ($\dot{x}(t)$)

Word-of-mouth effect (α_2)

Price of gasoline car (p_a)

Model: Firm's objective

- Maximize discounted profit over T

$$J^f = \max_{p(t)} \int_0^T e^{-\rho t}(p(t) - c(x(t)))\dot{x}(t)dt$$

- Cost to capture learning-by-doing:

$$c(x(t)) = b_1 - \mathbf{b_2}\, x(t)$$

↑ **Speed of learning**

Model: Government's problem

- Government: subsidy adjustment η_i at τ_i, $i = \{1, \cdots, N\}$

$$s\left(\tau_i^+\right) = s\left(\tau_i^-\right) + \boldsymbol{\eta_i}$$

Model: Government's problem

- Reach **target sales** x_s at $\tau_{N+1} < T$ with minimum expenditure
- **Fixed cost** associated with subsidy adjustments: C

$$J^g = \min_{\eta_i,\, x(\tau_{N+1}) \geq x_s} \left(\int_0^{\tau_{N+1}} e^{-\rho t} s(t) \dot{x}(t) dt + \sum_{i=1}^{N} e^{-\rho \tau_i} C \delta_{\eta_i > 0} \right)$$

Model: Stackelberg strategies

Government's Feedback strategy

$$\eta_i = \gamma^g(\tau_i, s(\tau_i), x(\tau_i))$$

Firm's Feedback strategy

$$p(t) = \gamma^f(t, s(t), x(t))$$

Model: Stackelberg strategies

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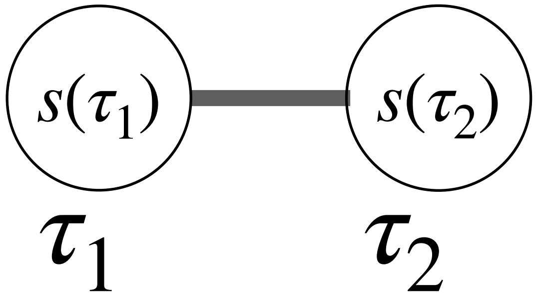
Announce η_1



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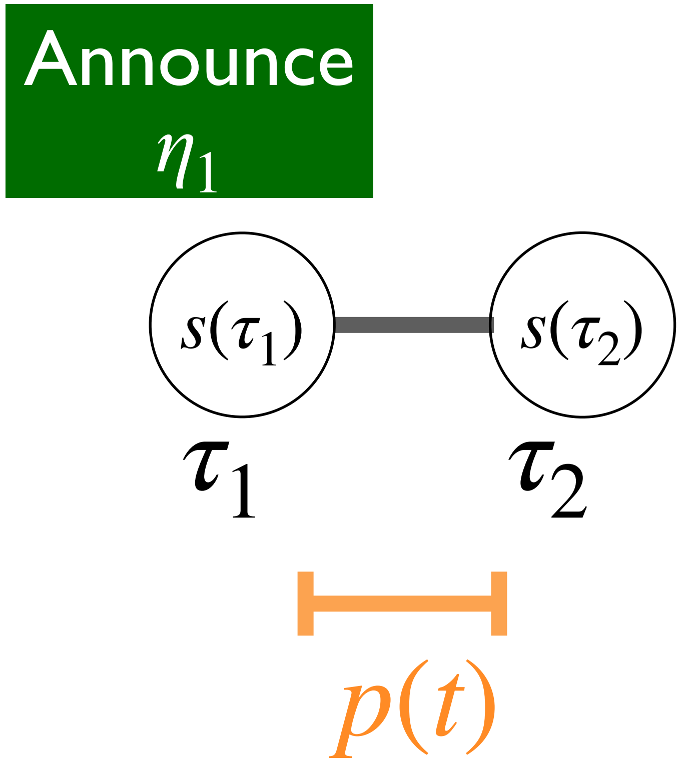

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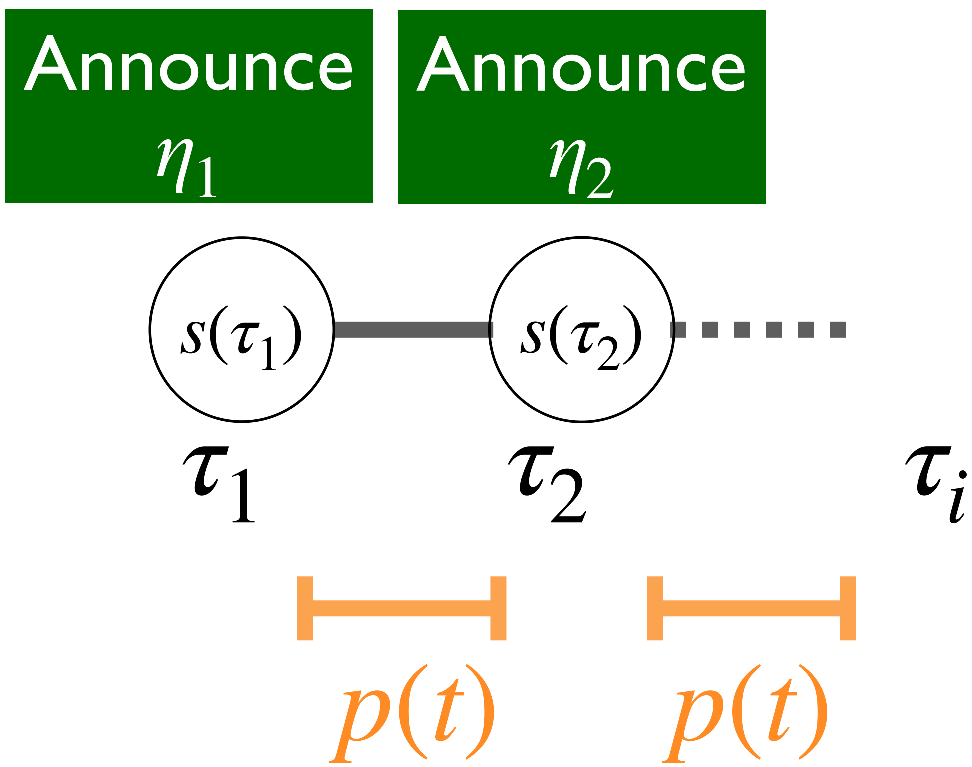

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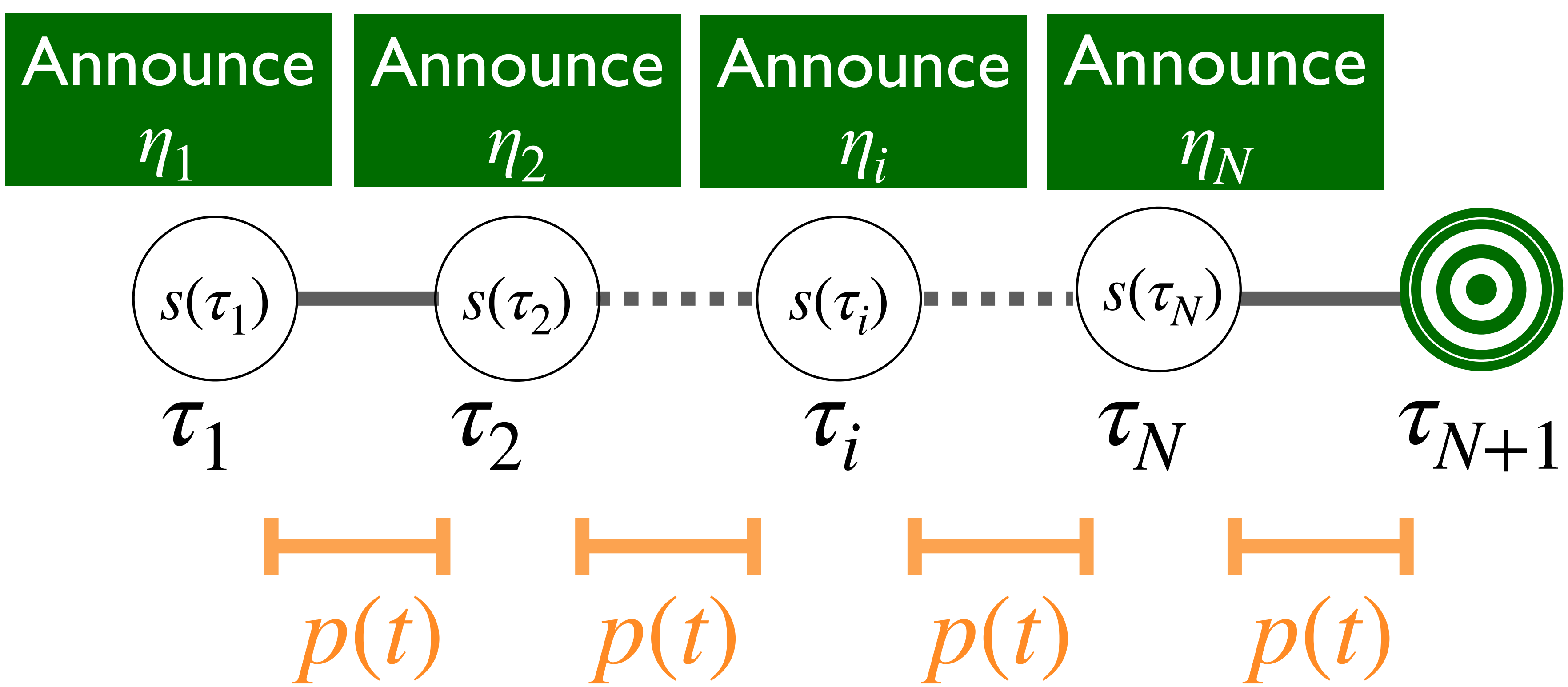

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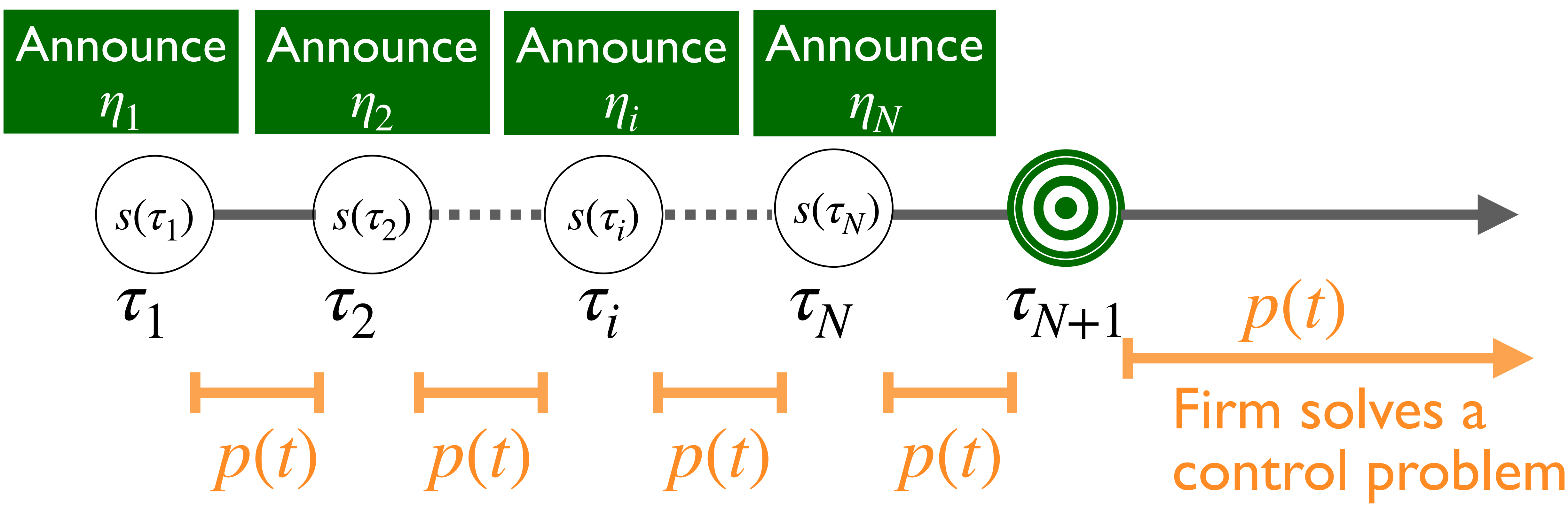

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Feedback Stackelberg equilibrium

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  - Government announces strategy γ^g
  - $\hat{\gamma}^f(\cdot, \gamma^g)$ is firm's best response to γ^g
  - $\hat{\gamma}^g$ minimizes the government's cost given best response $\hat{\gamma}^f(\cdot, \hat{\gamma}^g)$
  - Pair $(\hat{\gamma}^g, \hat{\gamma}^f)$ constitutes the FSE of the game

Solving the game

Firm’s problem

Computing equilibria: Firm's problem

HJB equation

$$\rho v^f(t,x) - v_t^f(x) = \max_{p(t)} \Big[(p(t) - c(x(t)) + v_x^f(t,x)) \times (\alpha_1 + \alpha_2 x(t) - \beta(p(t) - p_a)) \Big]$$

Computing equilibria: Firm's problem

- After the target date τ_{N+1}, firm solves LQ control problem

HJB equation

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Computing equilibria: Firm's problem

- After the target date τ_{N+1}, firm solves LQ control problem
- Subsidy is 0 in this region

HJB equation

$$\rho v^f(t,x) - v_t^f(x) = \max_{p(t)} \Big[(p(t) - c(x(t)) + v_x^f(t,x)) \\ \times (\alpha_1 + \alpha_2 x(t) - \beta(p(t) - p_a)) \Big]$$

Firm's problem

- Value function of the firm satisfies Hamilton-Jacobi-Bellman equation (HJB)
- How to solve HJB equation? Infinite dimensional problem
- Search for value functions in space of quadratic (in state) functions

$$v^f(t,x) = \frac{1}{2}k_2(t)x^2 + k_1(t)x + k_0(t)$$

Ricatti system

$$\rho k_2(t) - \dot{k}_2(t) = \frac{\beta}{2}\left(\frac{w_2}{\beta} + k_2(t)\right)^2$$

$$\rho k_1(t) - \dot{k}_1(t) = \frac{\beta}{2}\left(\frac{w_1}{\beta} + k_1(t)\right)\left(\frac{w_2}{\beta} + k_2(t)\right)$$

$$\rho k_0(t) - \dot{k}_0(t) = \frac{\beta}{4}\left(\frac{w_1}{\beta} + k_1(t)\right)^2$$

Between decision dates

Between decision dates

- Between consecutive decision dates τ_i and τ_{i+1} for $i \in \{0, 1, \cdots, N\}$

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Between decision dates

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  - government does not act
  - Value function of the firm satisfies Hamilton-Jacobi-Bellman equation
  - Quadratic (in-the-state) value function

Ricatti system: between impulse dates

- **Value function depends on the subsidy level**

$$\tau_i^+ \leq t \leq \tau_{i+1}^-$$

$$\rho k_2(t) - \dot{k}_2(t) = \frac{\beta}{2}\left(\frac{w_2}{\beta} + k_2(t)\right)^2$$

$$\rho k_1(t) - \dot{k}_1(t) = \frac{\beta}{2}\left(\frac{w_1}{\beta} + k_1(t) + s(\tau_i^+)\right)\left(\frac{w_2}{\beta} + k_2(t)\right)$$

$$\rho k_0(t) - \dot{k}_0(t) = \frac{\beta}{4}\left(\frac{w_1}{\beta} + s(\tau_i^+) + k_1(t)\right)^2$$

..at impulse dates

- Value function of the firm is **continuous** at the impulse date
- However, a change of subsidy introduces **kinks** in the value function

$$k_2(\tau_i^+) = k_2(\tau_i^-)$$
$$k_1(\tau_i^+) = k_1(\tau_i^-)$$
$$k_0(\tau_i^+) = k_0(\tau_i^-)$$

Government's problem

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- Government has the **target** to reach at least state x_s by time τ_{N+1}
- If target is not reached $\Rightarrow$ infinite penalty
- v^g: value function of the government

Government's problem

- Government has the **target** to reach at least state x_s by time τ_{N+1}
- If target is not reached $\Rightarrow$ infinite penalty
- $\boldsymbol{v^g}$: value function of the government

Terminal condition

$$v^g(\tau_{N+1}, s(\tau_{N+1}), x(\tau_{N+1})) = \begin{cases} 0, & \text{if } x(\tau_{N+1}) \geq x_s \\ \infty, & \text{otherwise.} \end{cases}$$

Government's problem

Minimum cost

$$\mathscr{M} v^g(\tau_i, s, x(\tau_i))$$

$$= \min_{\eta_i \in \Omega^g(s)} \Bigg(\int_{\tau_i^+}^{\tau_{i+1}^-} e^{-\rho t}(s + \eta_i)\dot{x}(t)dt$$

[Cost of subsidy]

$$+ C\delta_{\eta_i > 0}$$

[Fixed cost]

$$+ v^g(\tau_{i+1}, s + \eta_i, x(\tau_{i+1})) \Bigg)$$

[Optimal cost from playing optimally afterwards]

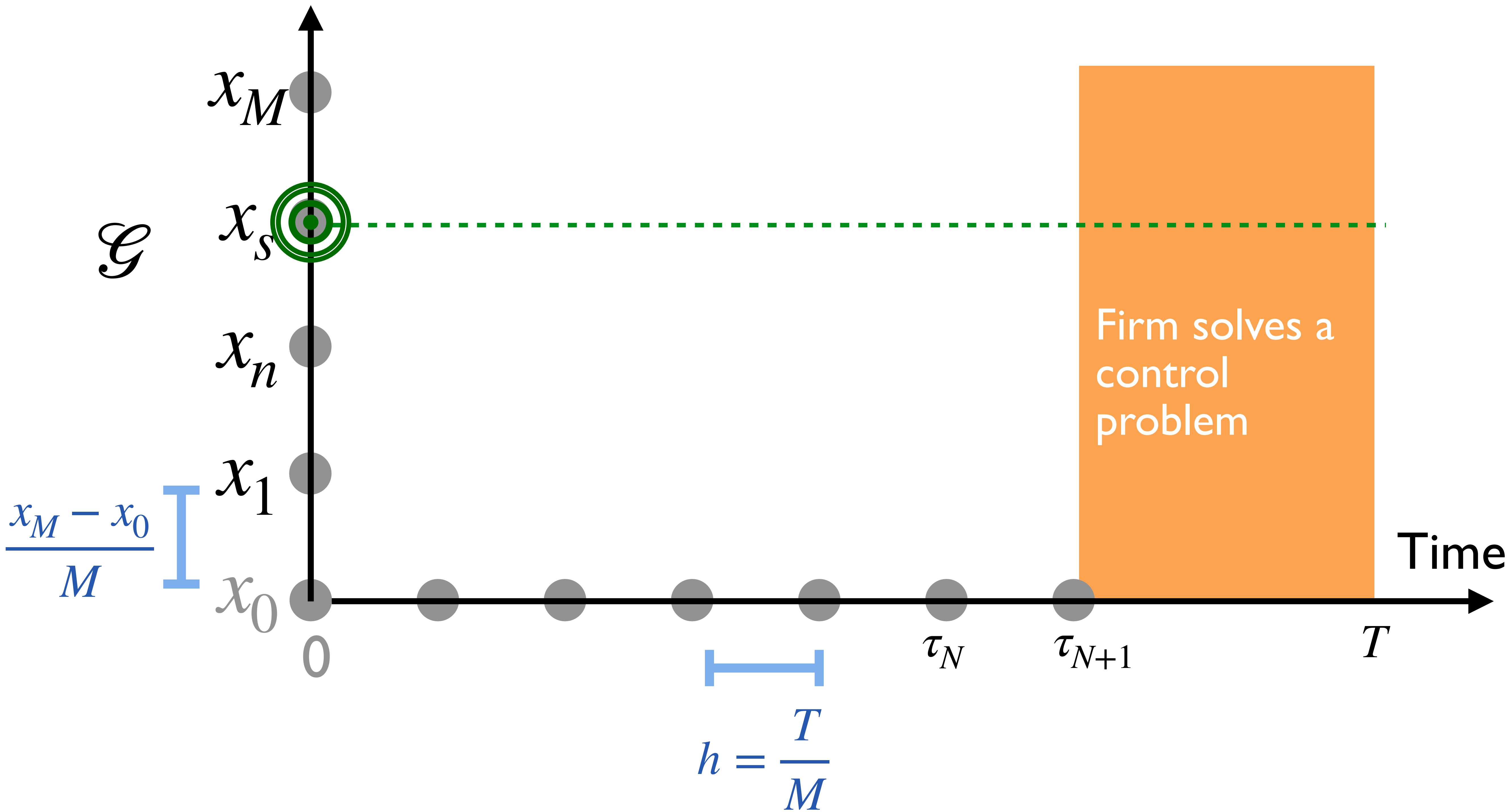
subject to firm's optimal response

Government's problem

- Value function should be equal to the minimum cost that can be achieved by government by playing optimally

$$v^g(\tau_i, s, x(\tau_i)) = \mathscr{M} v^g(\tau_i, s, x(\tau_i))$$

Algorithm to compute subsidy



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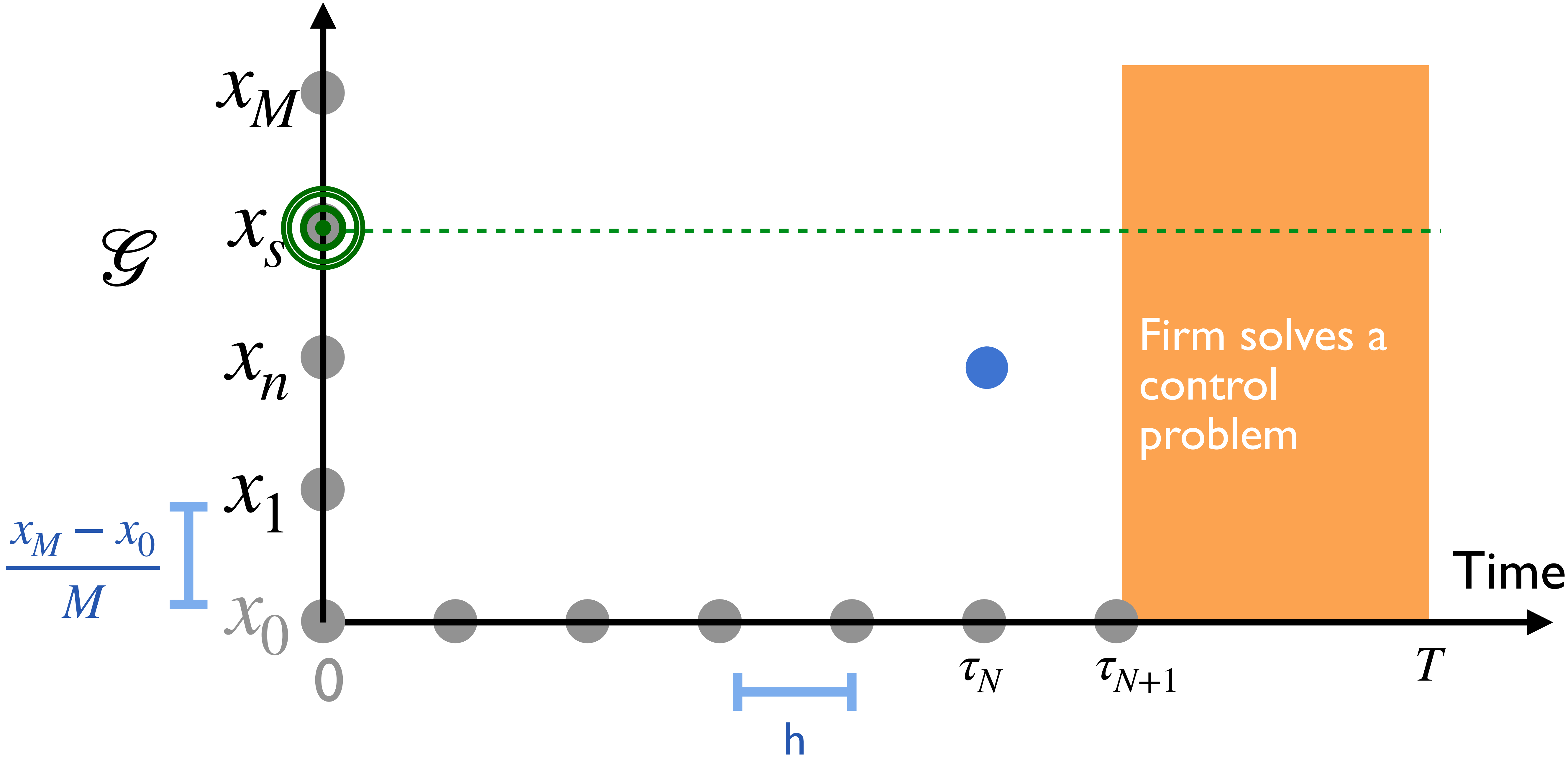

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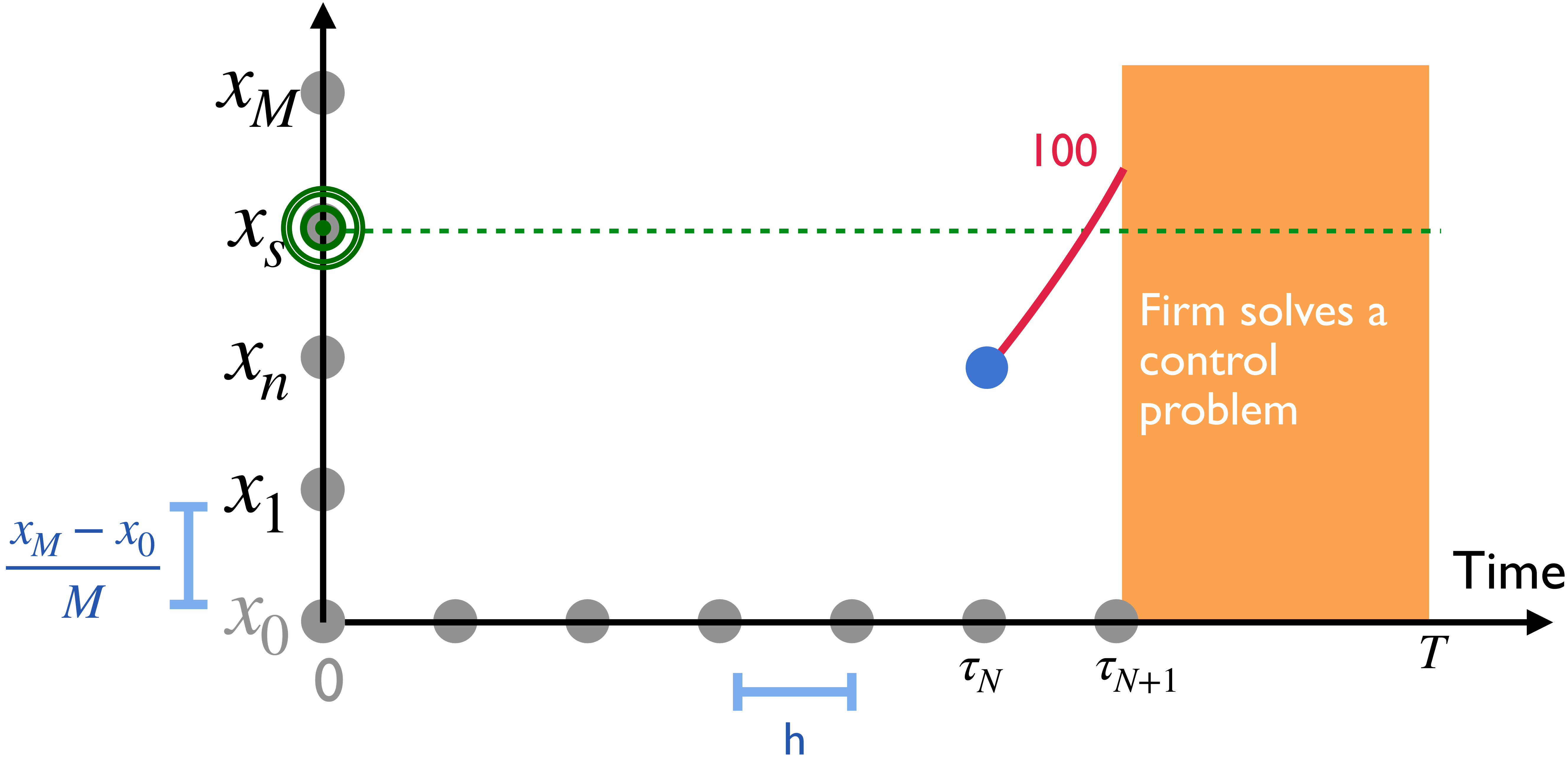

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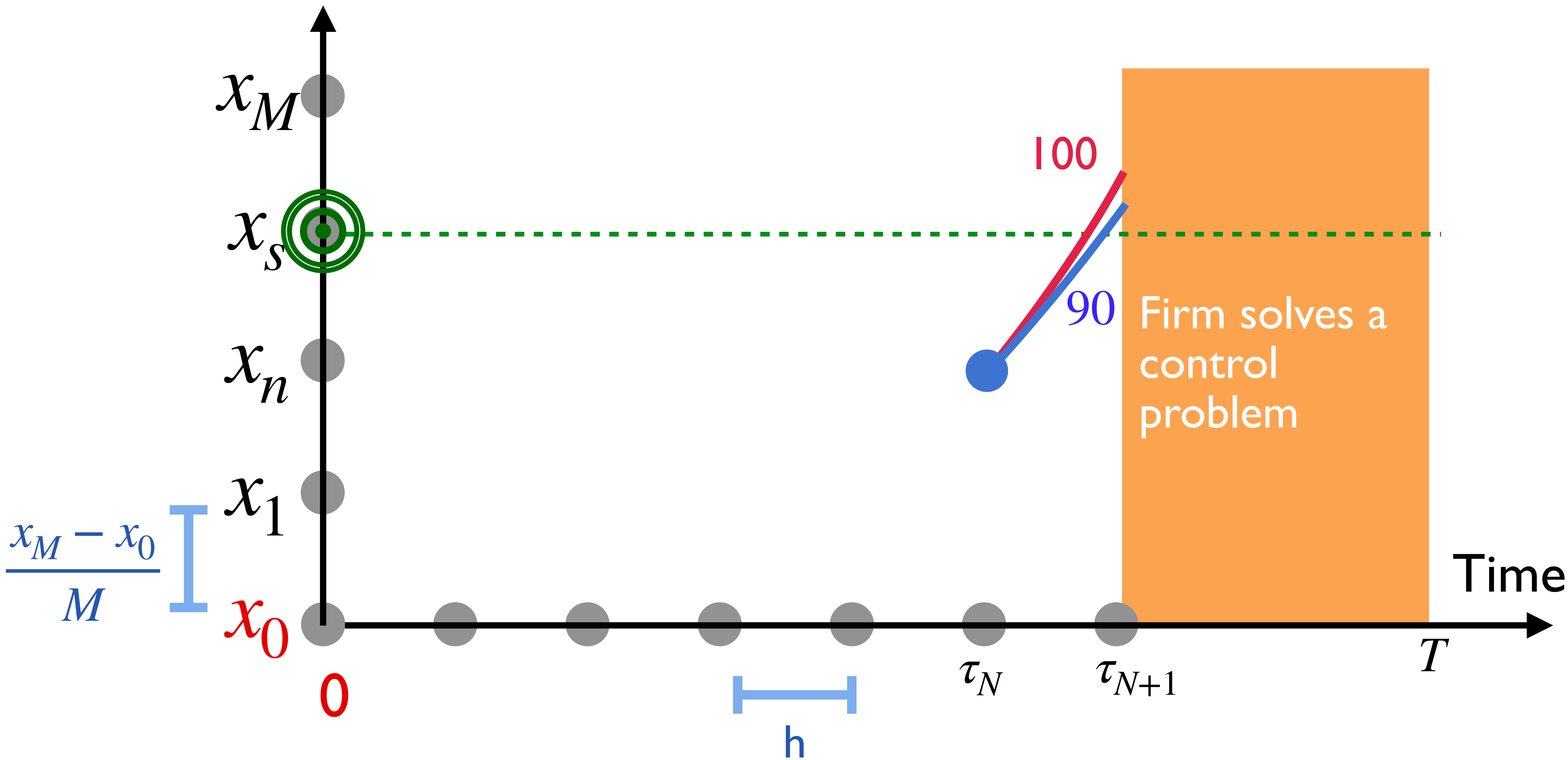

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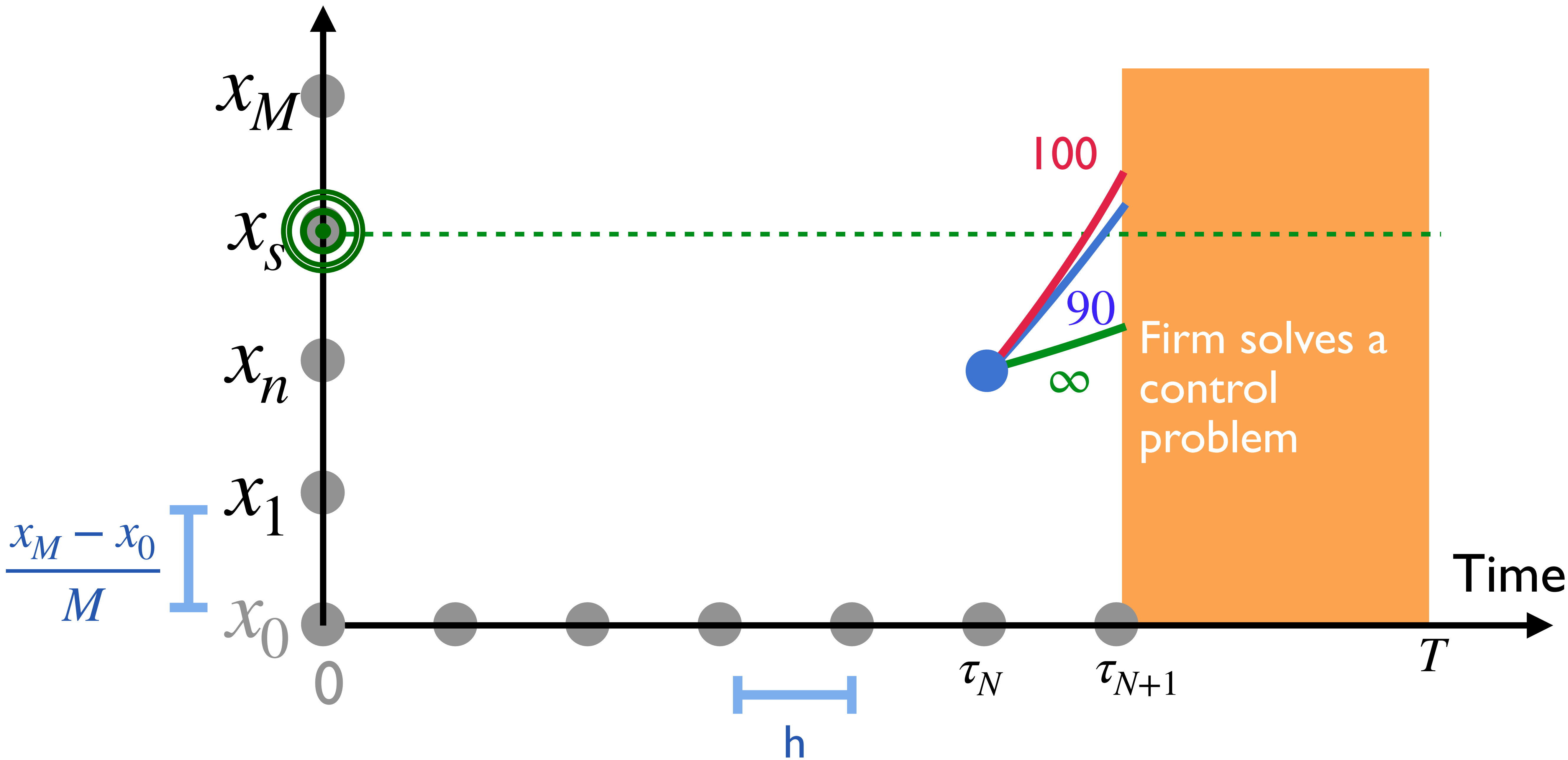

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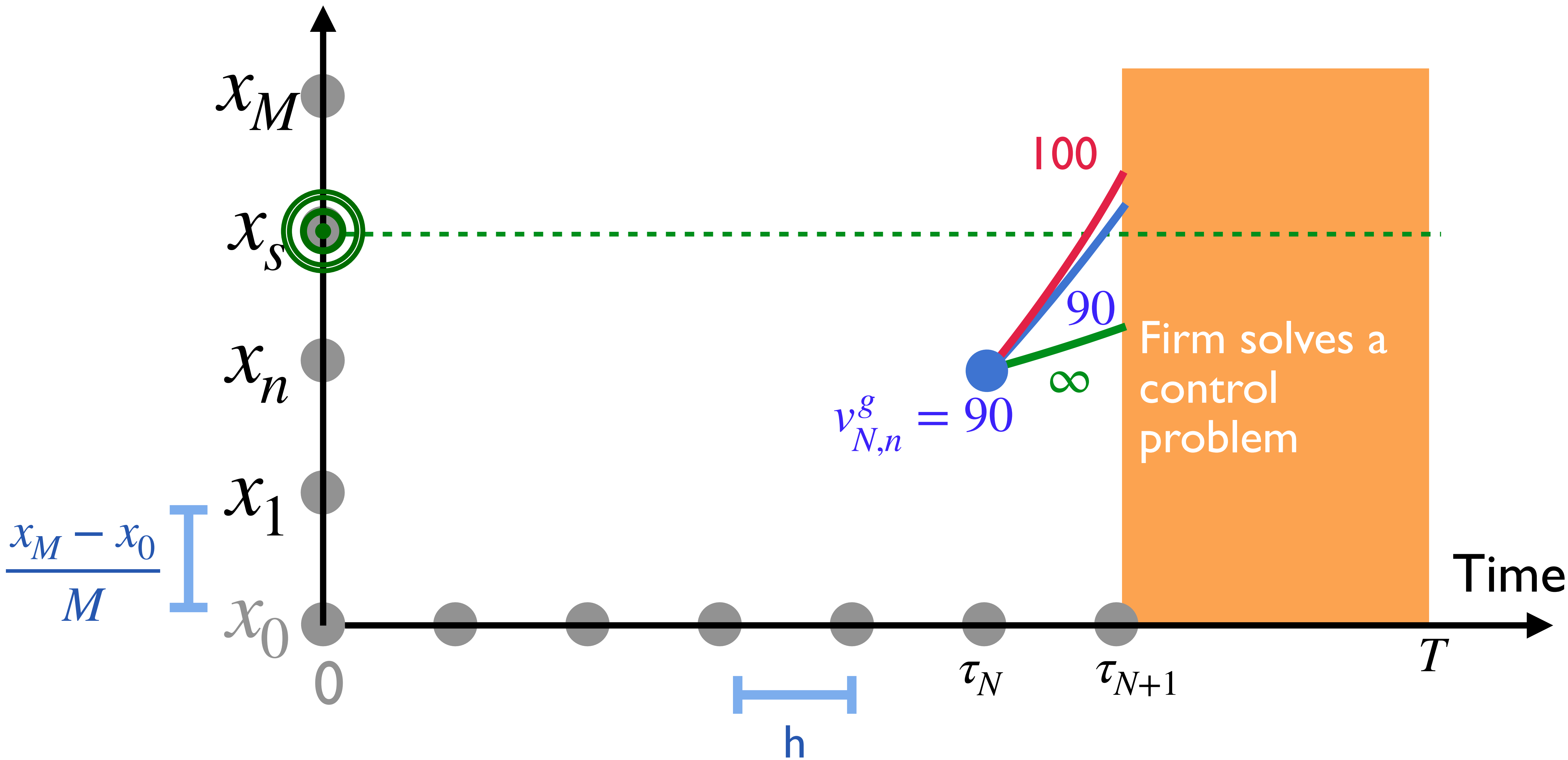

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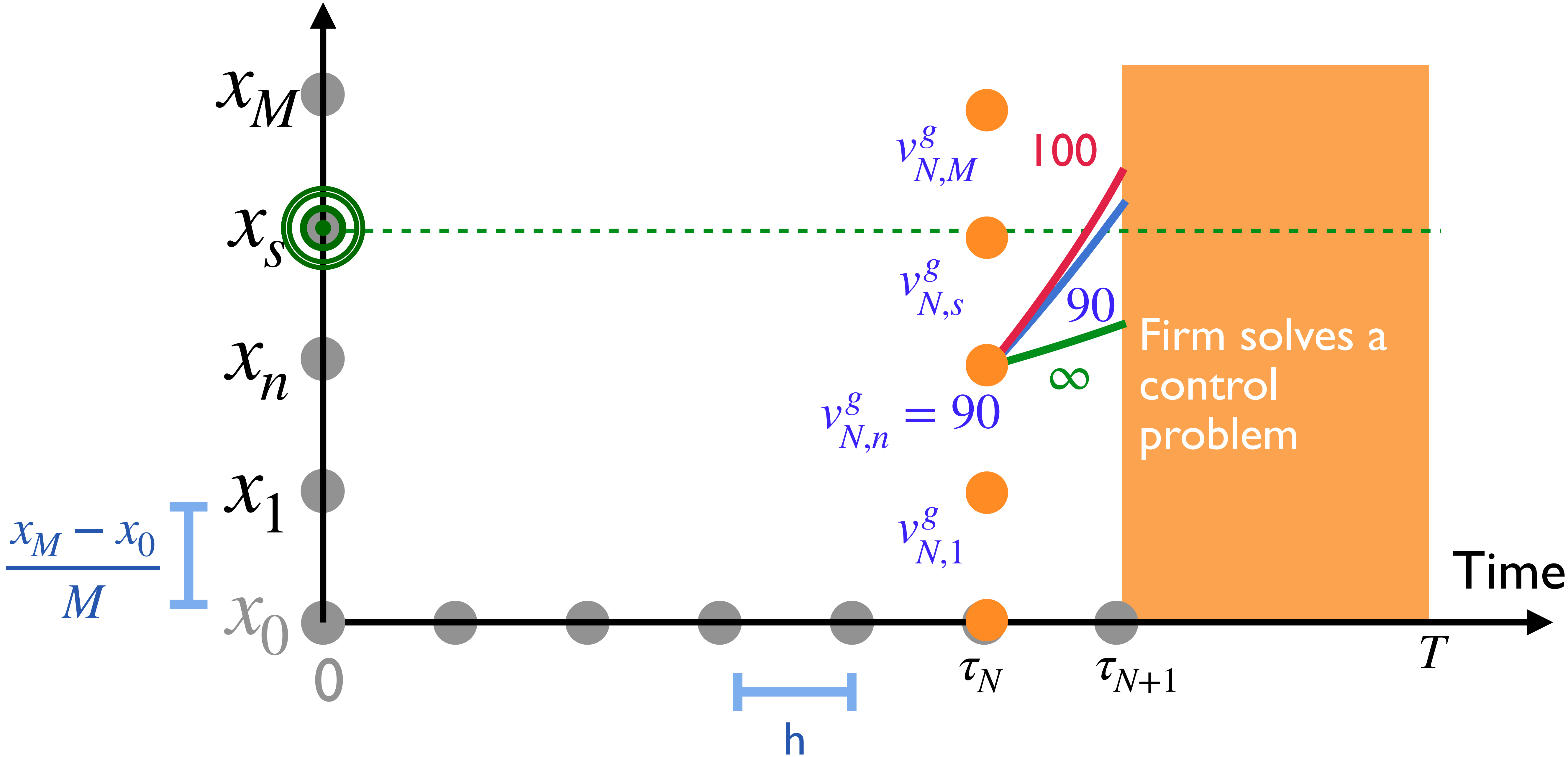

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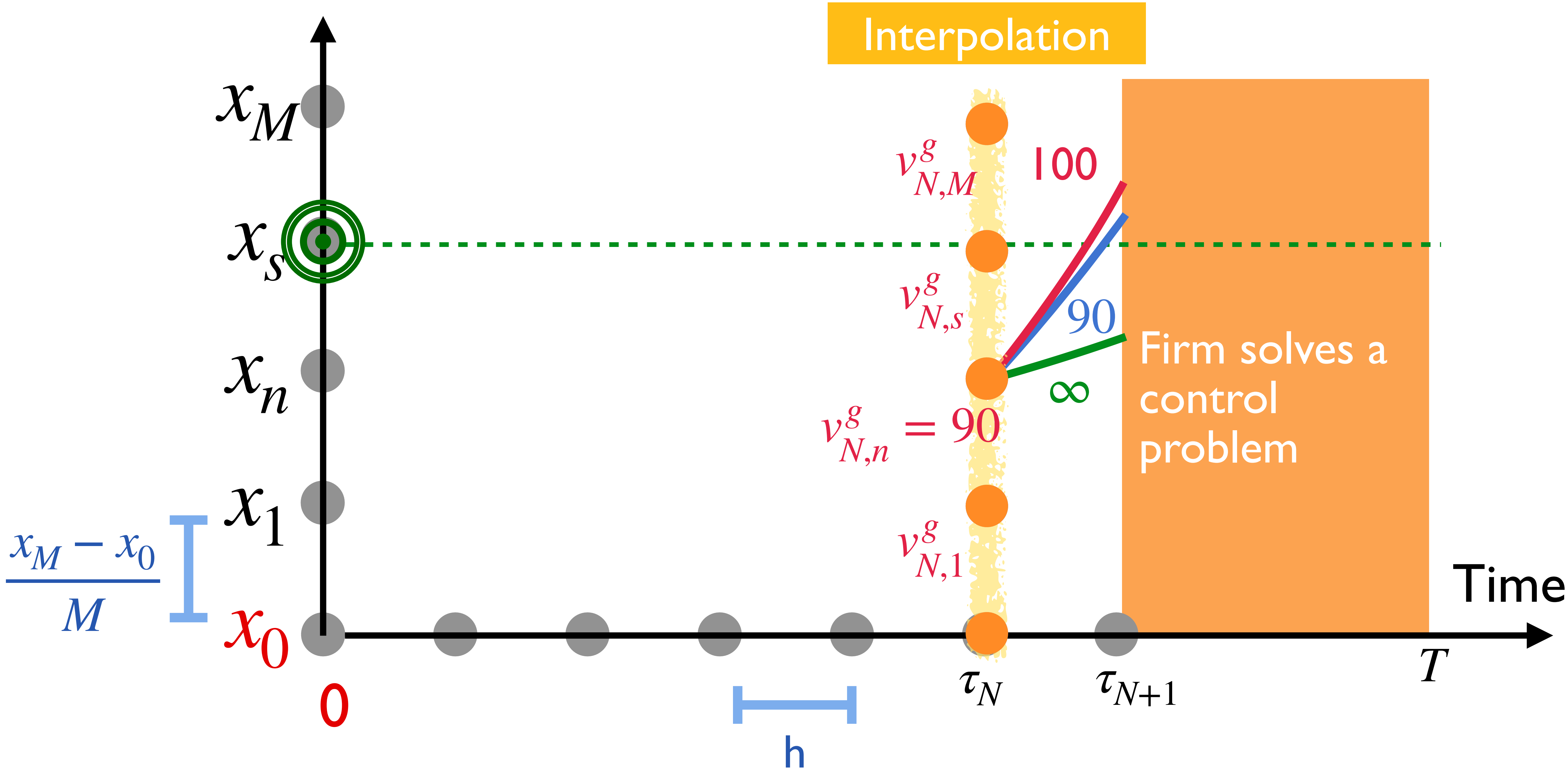

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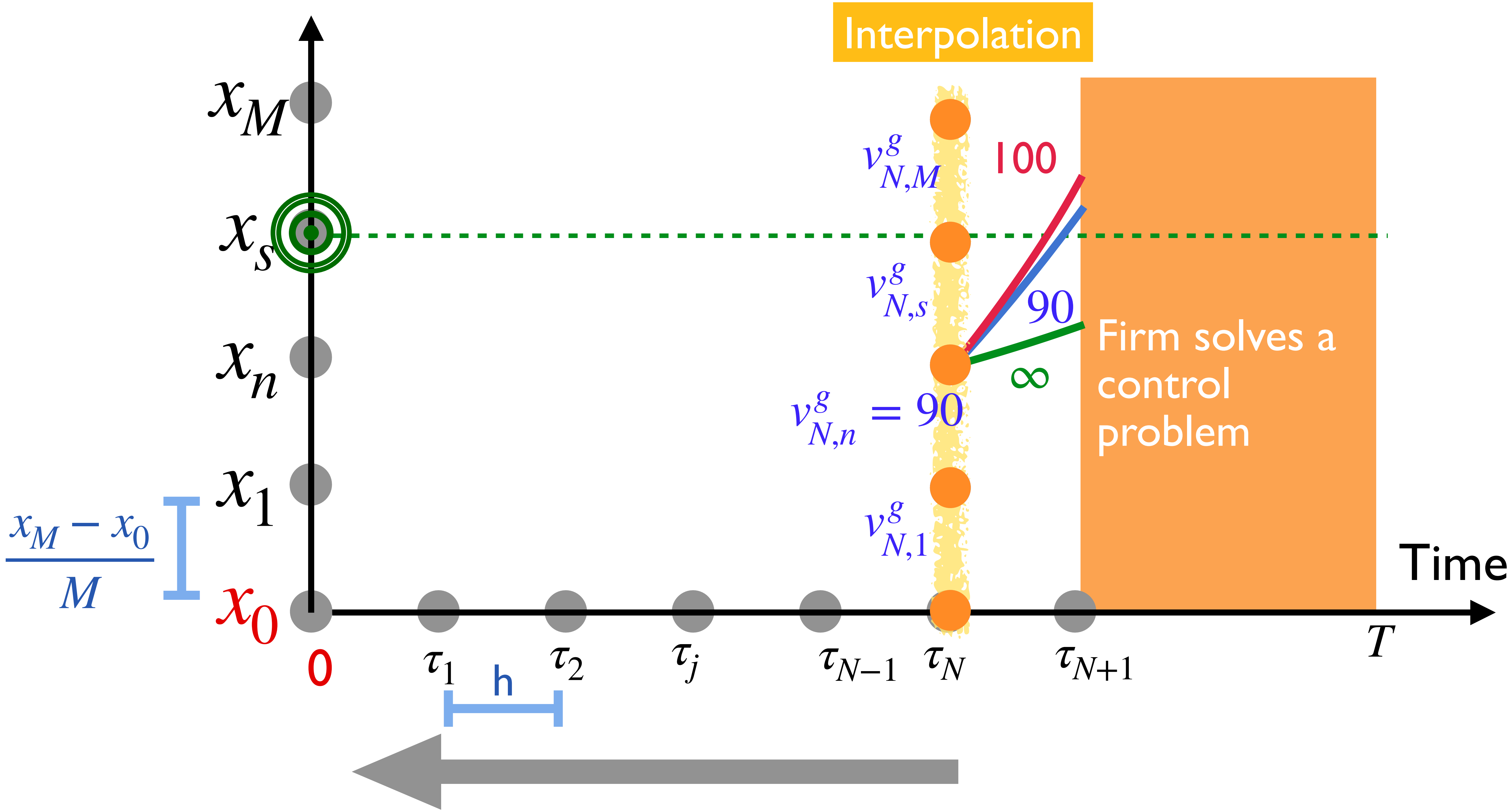


Numerical example

Speed of learning b_2

$$J^f = \max_{p(t)} \int_0^{18} e^{-0.1t}(p(t) - (50 - 0.8x(t)))\dot{x}(t)dt$$

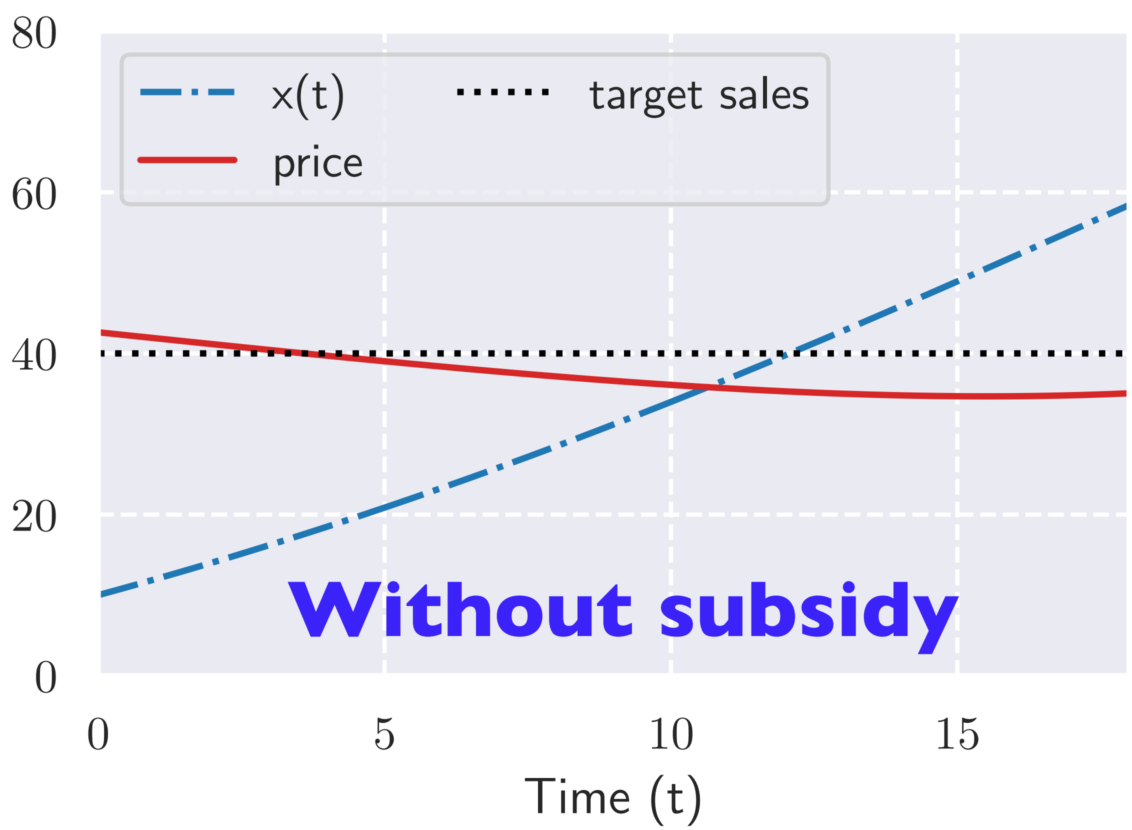
$$J^g = \min_{\eta_i} \left(\int_0^{10} e^{-0.1t} s(t)\dot{x}(t)dt + \sum_{i=1}^{2} e^{-\rho\tau_i} 10\delta_{\eta_i > 0} \right)$$

$$\dot{x}(t) = 6 + 0.01x(t) - 0.1(p(t) - s(t) - 1)$$

Word-of-mouth α_2

$$s\left(\tau_i^+\right) = s\left(\tau_i^-\right) + \eta_i \quad \text{for } i = \{1,2,.....,N\}$$

Target = 40 by time 10



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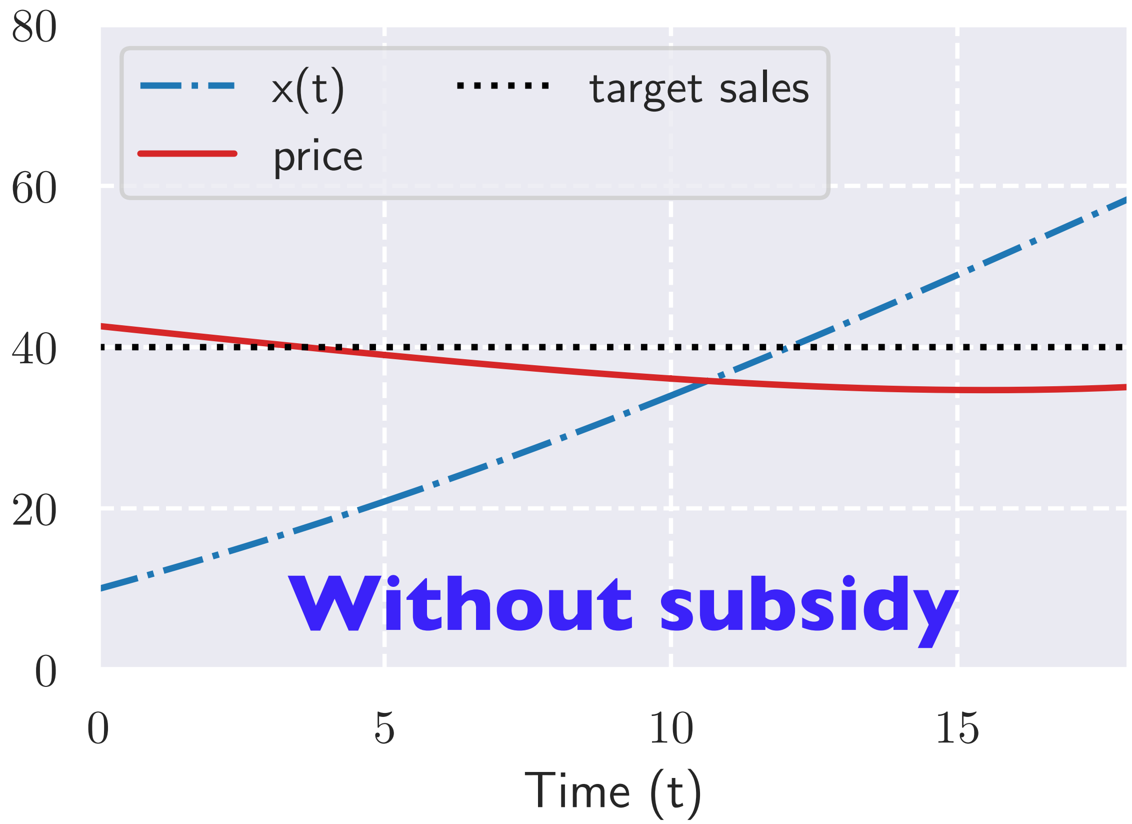

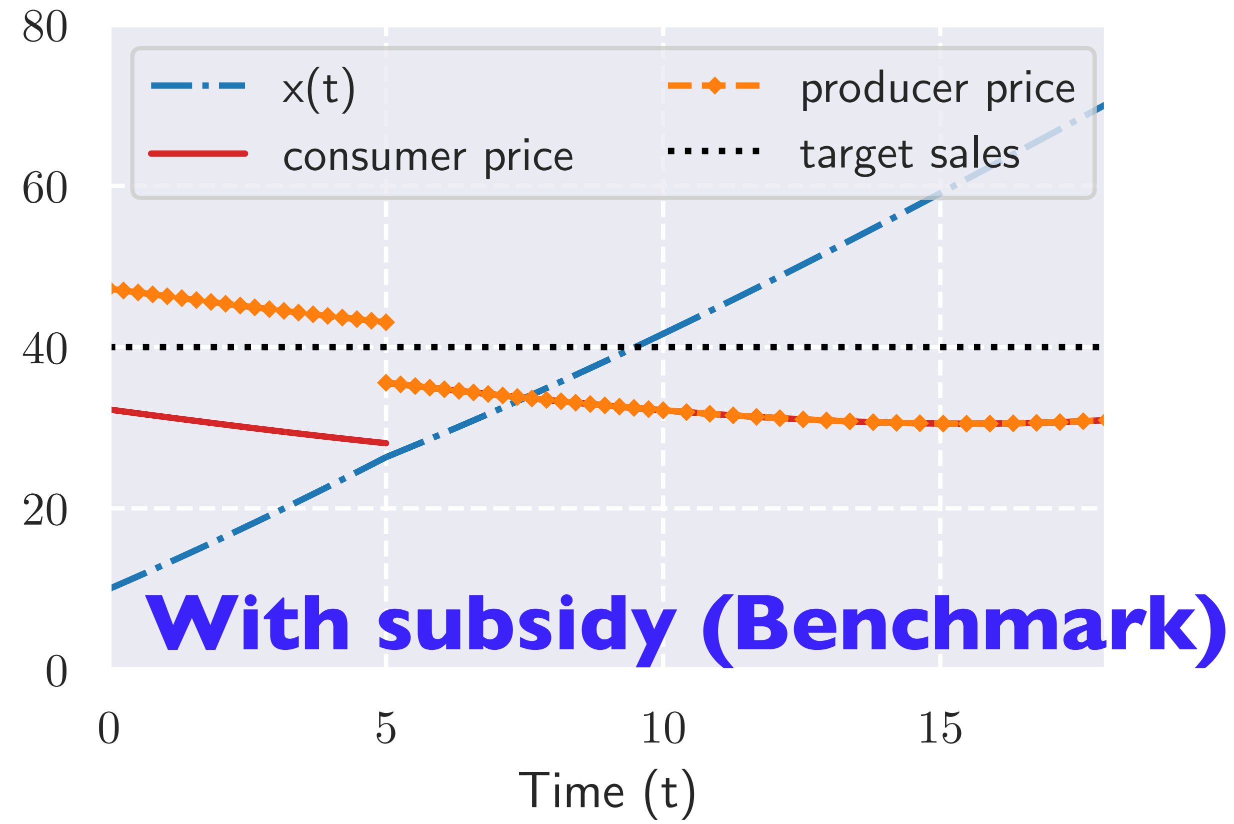

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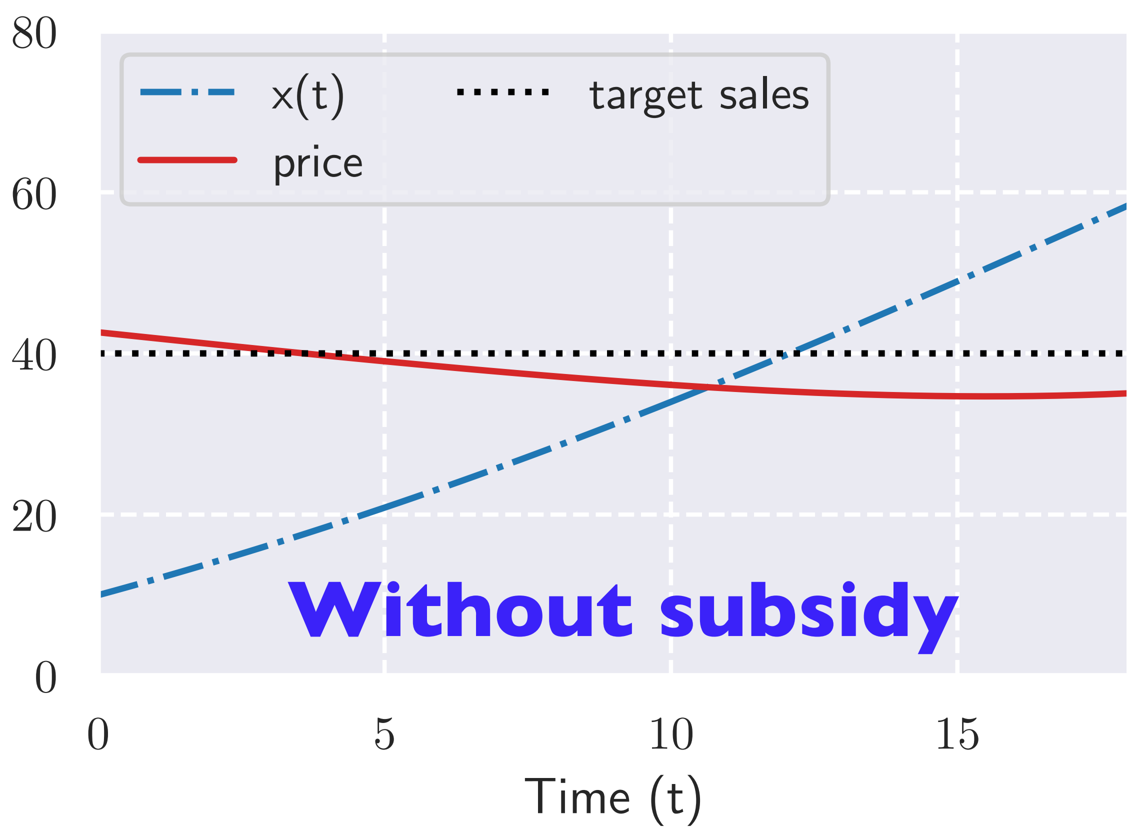
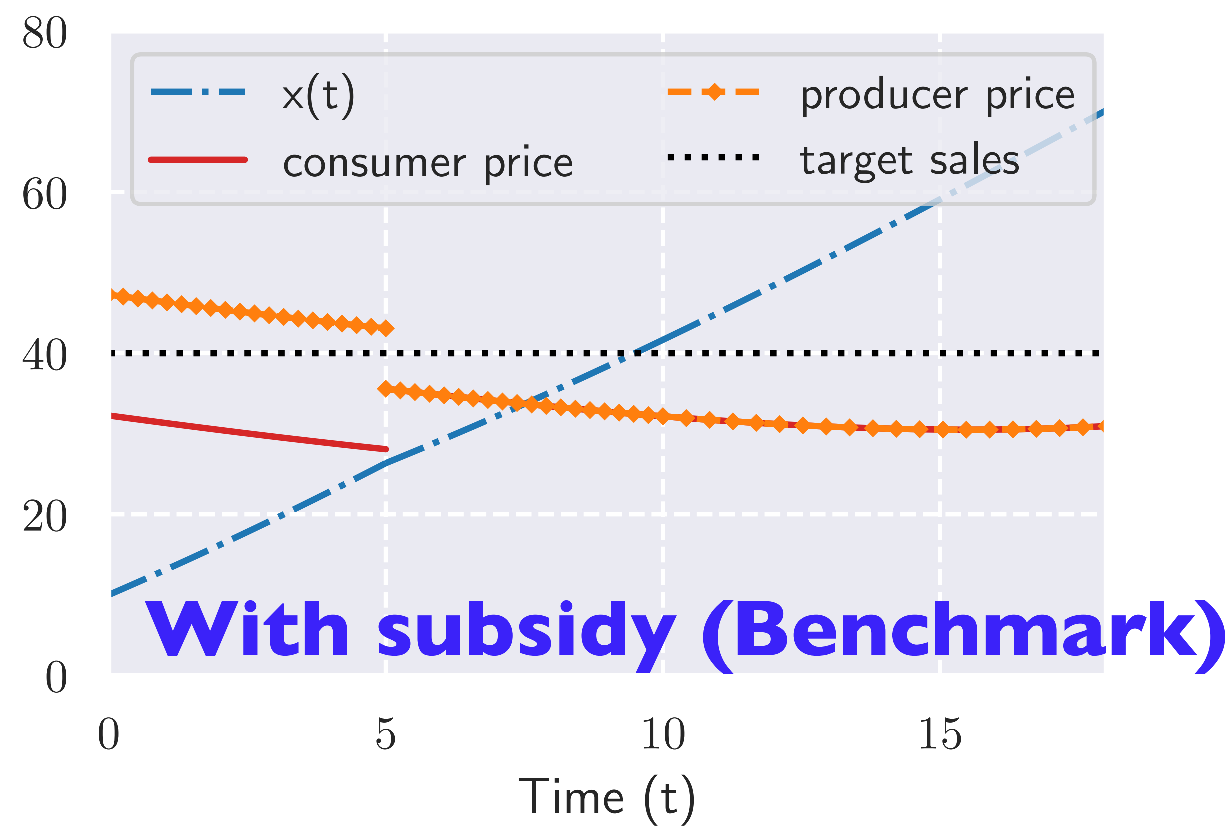
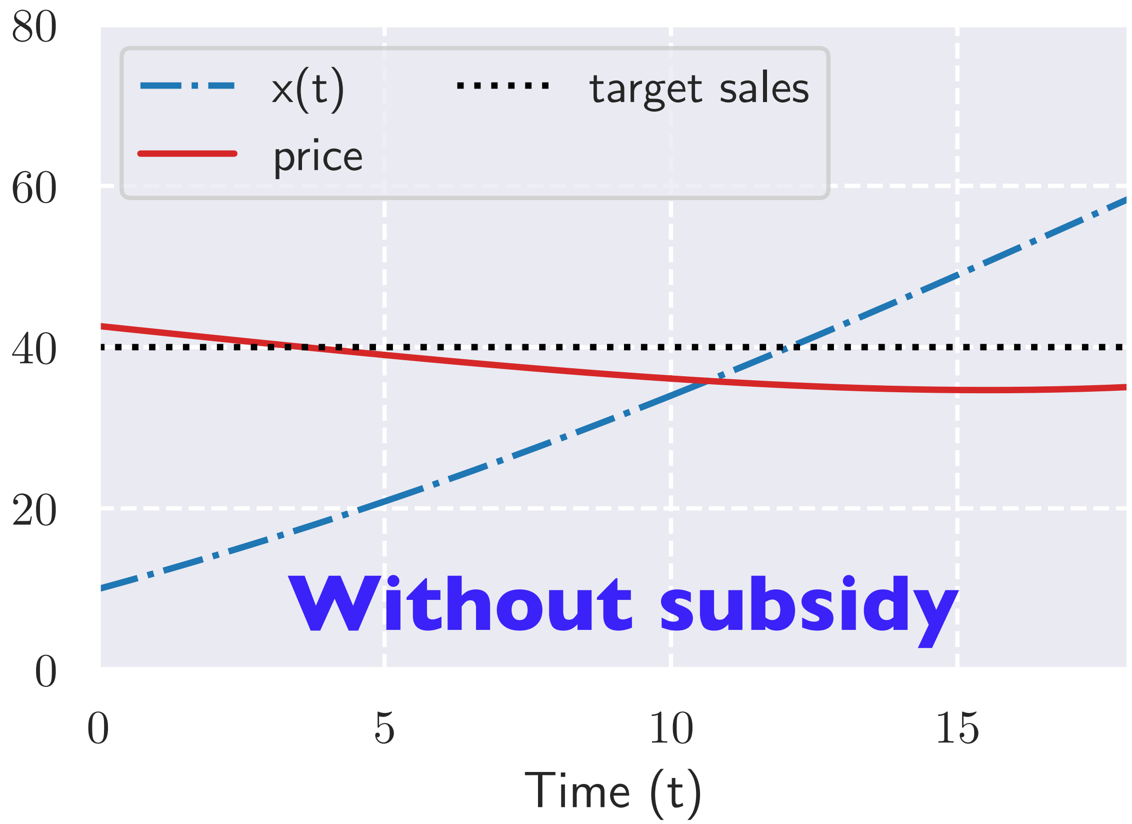
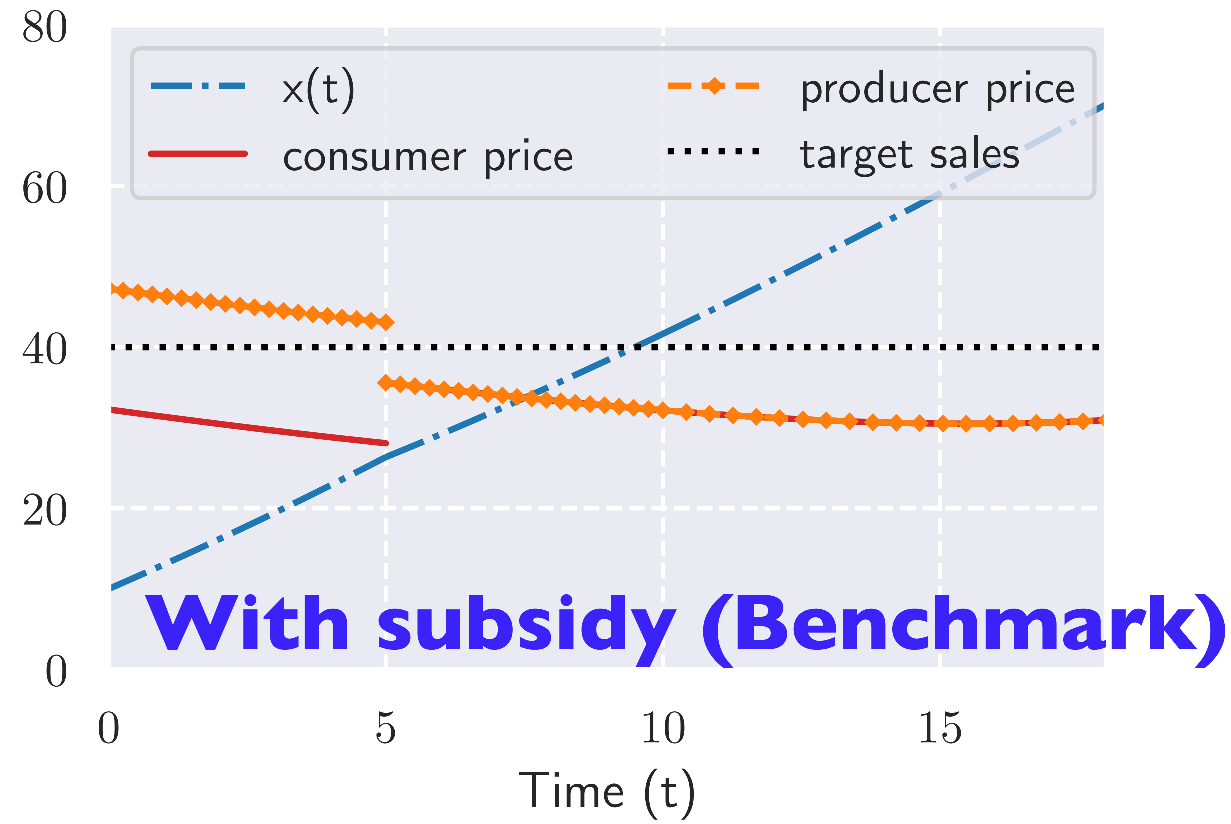




Target = 40 by time 10

- With subsidy
  - Firm's profit increase by 56%
  - Low price
  - High adoption
  - Target is met
  - Lower pollution





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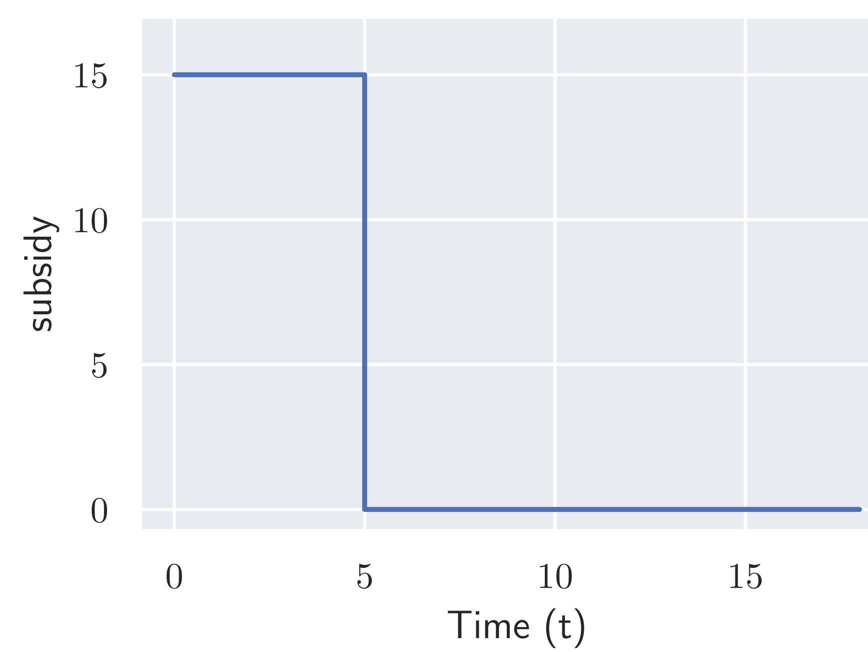
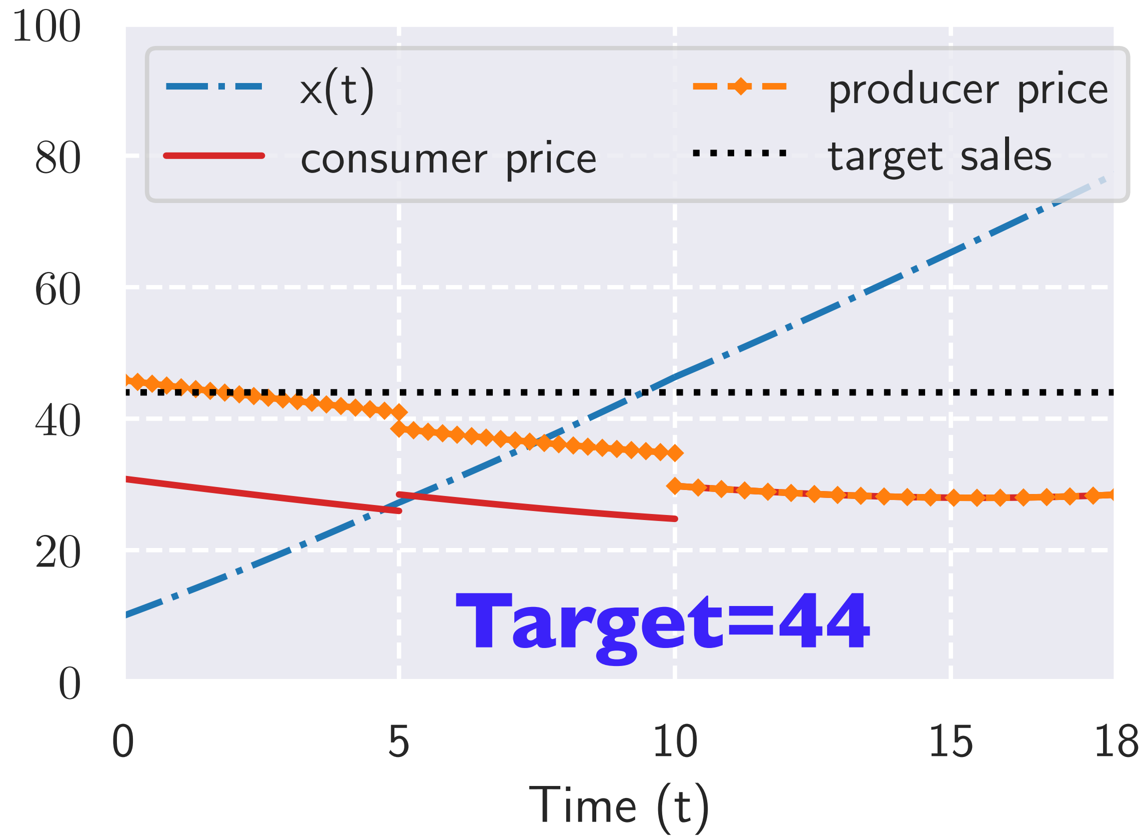
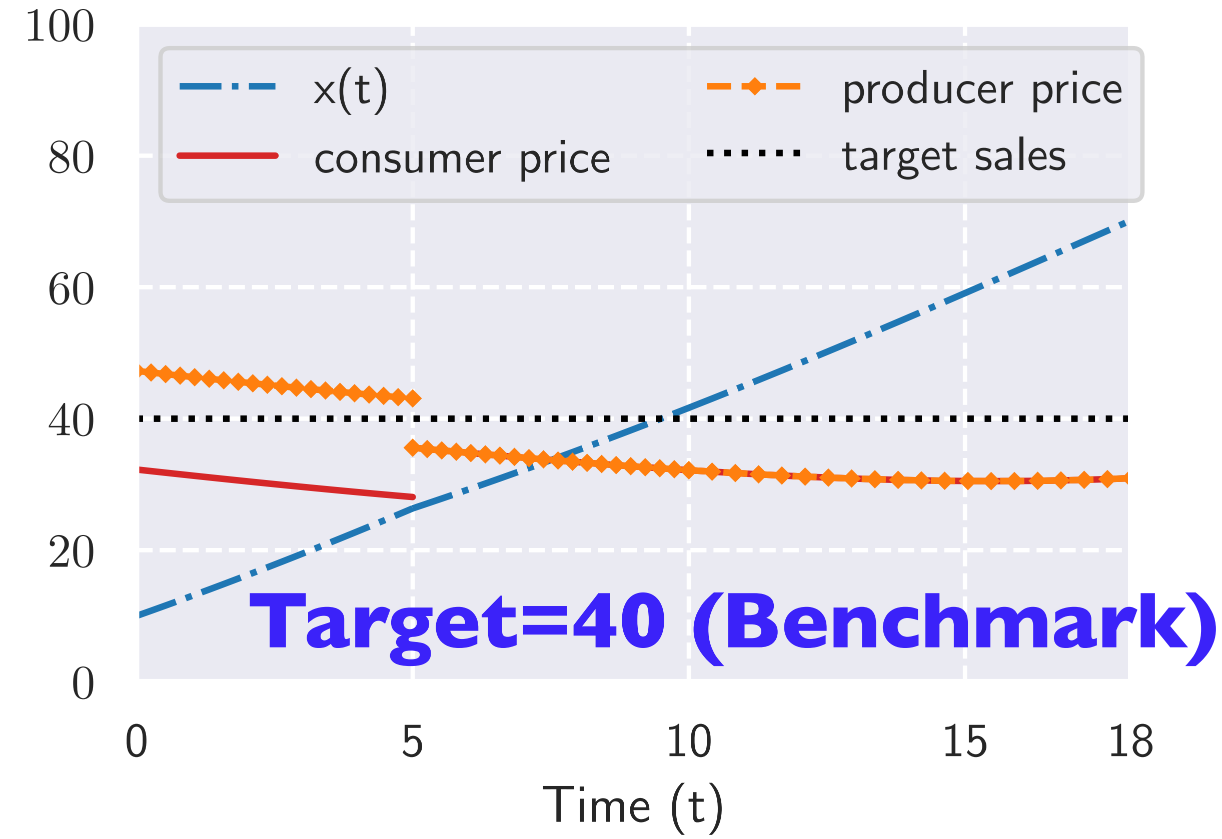


Impact of target value





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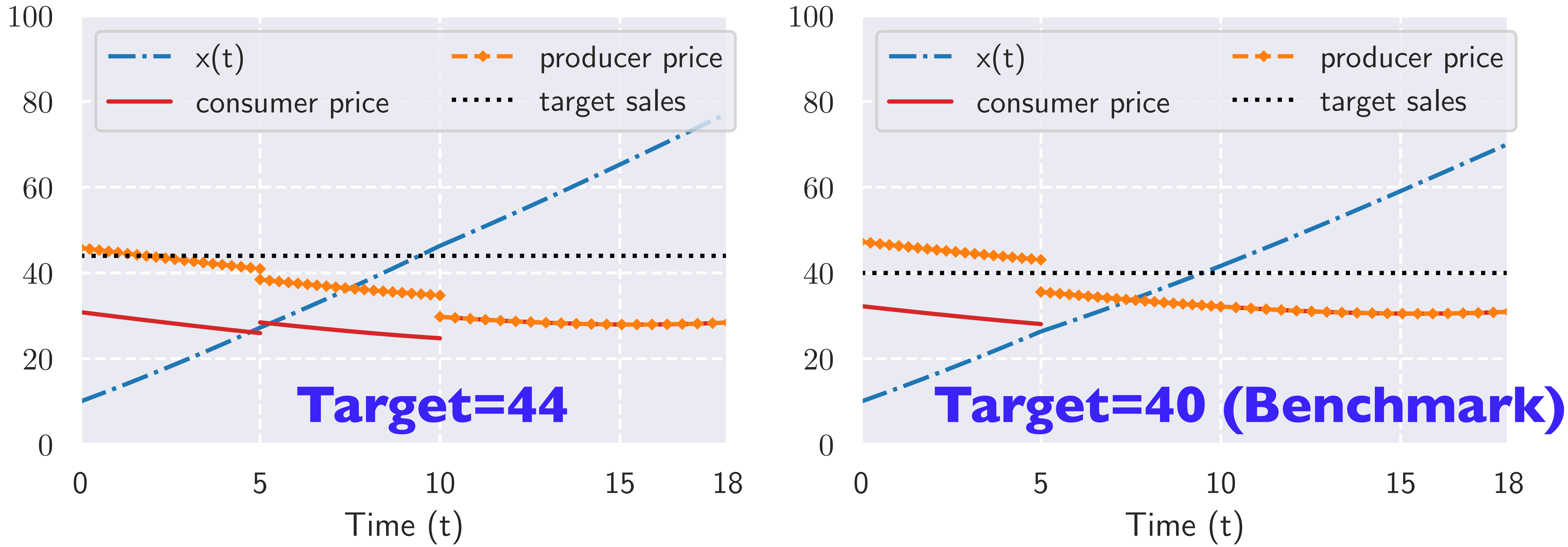
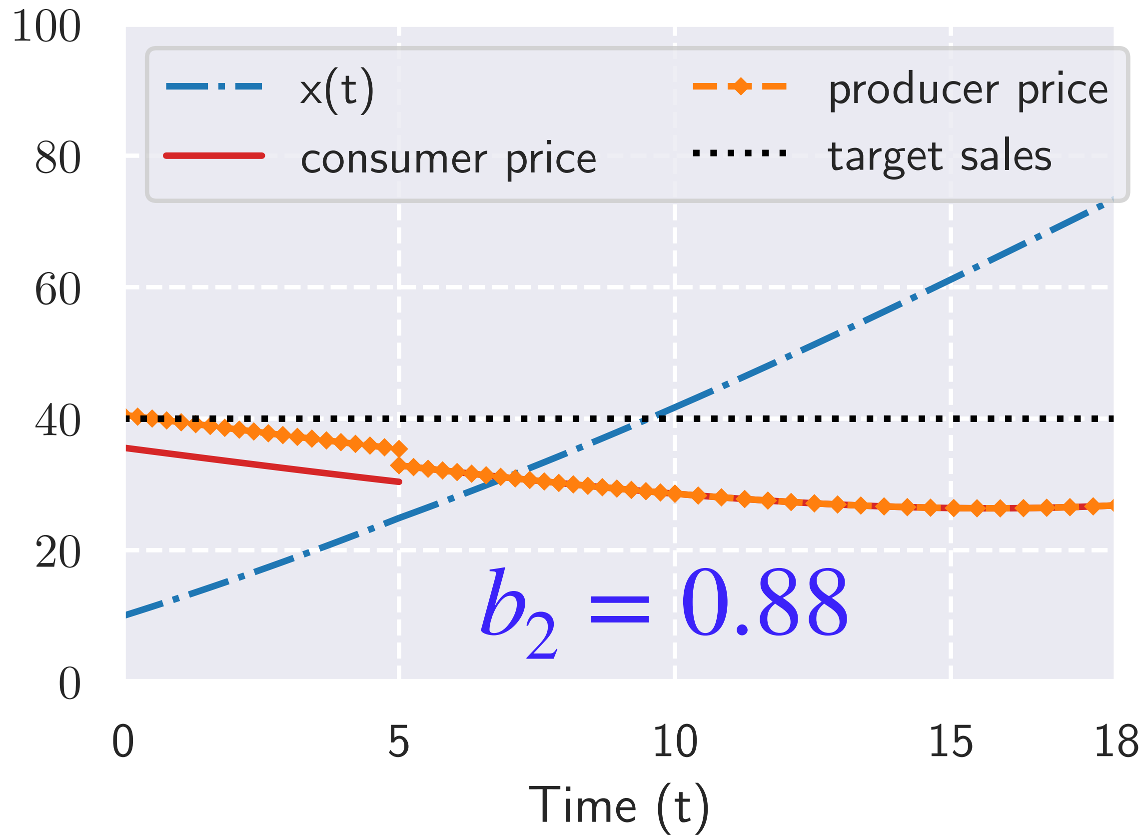
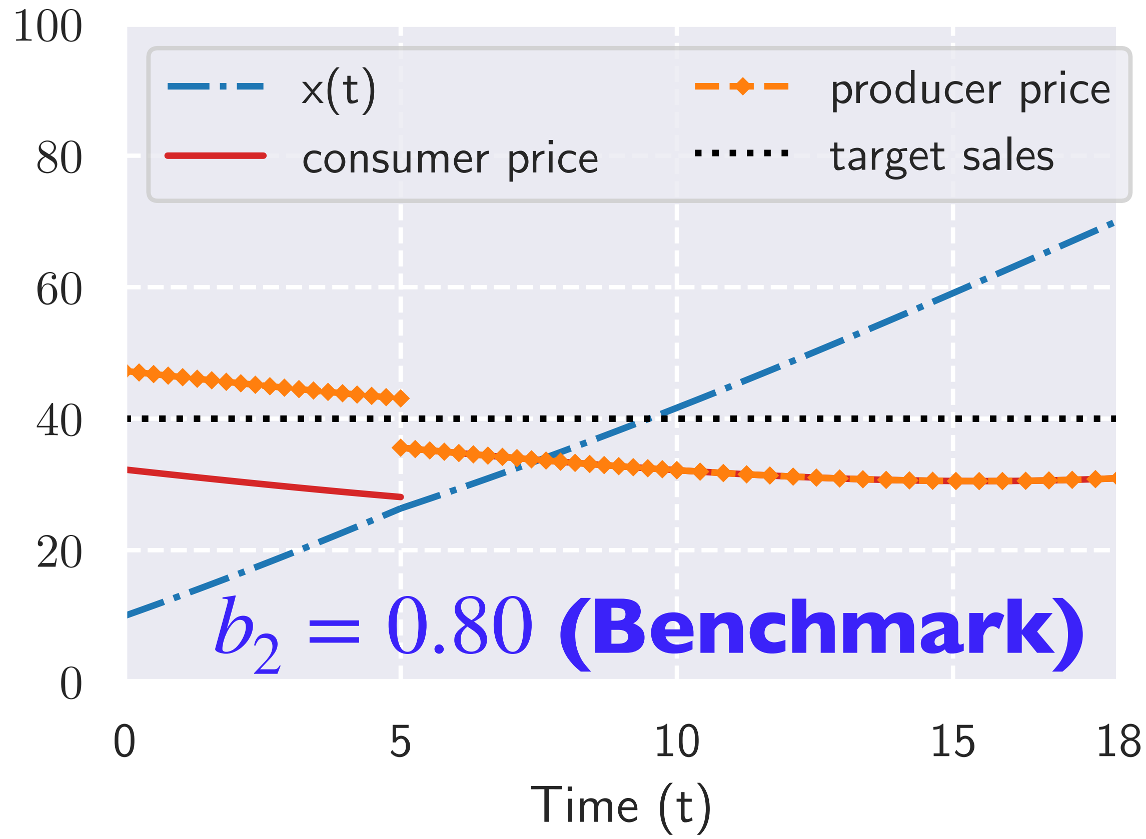


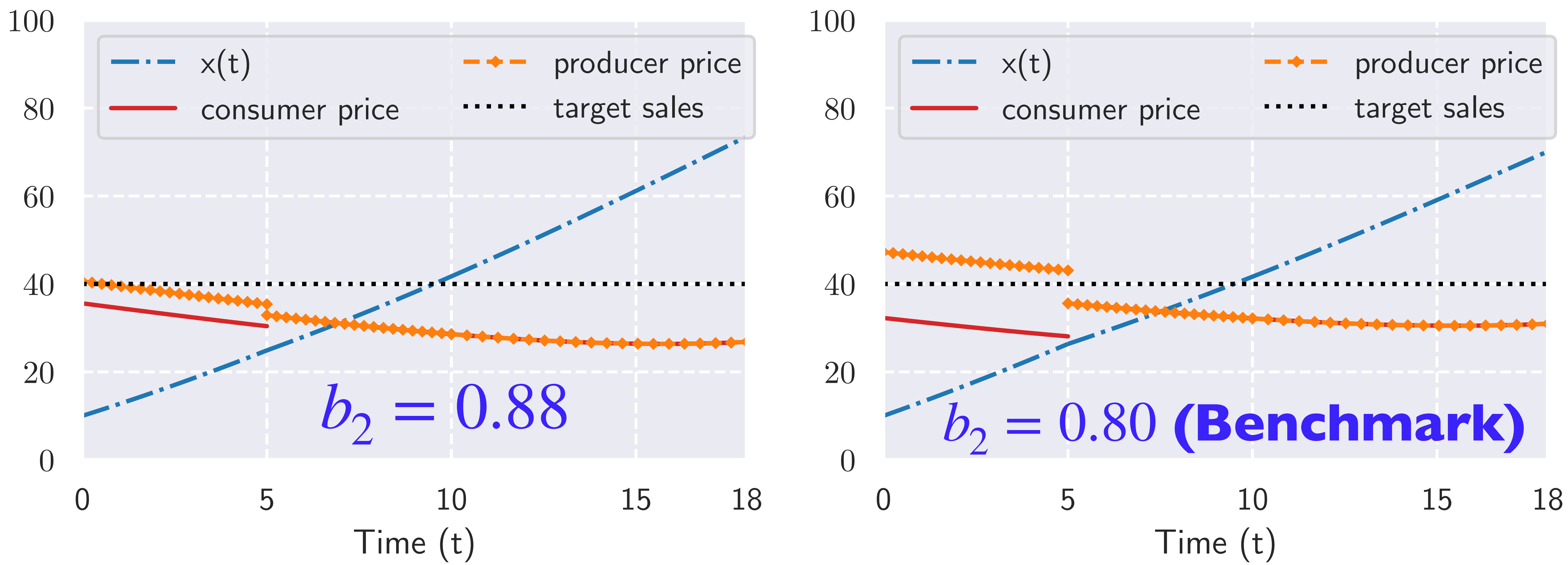
- Lower price
- Higher adoption
- Increase in $300\ \%$ in the cost to the taxpayers

Impact of learning speed



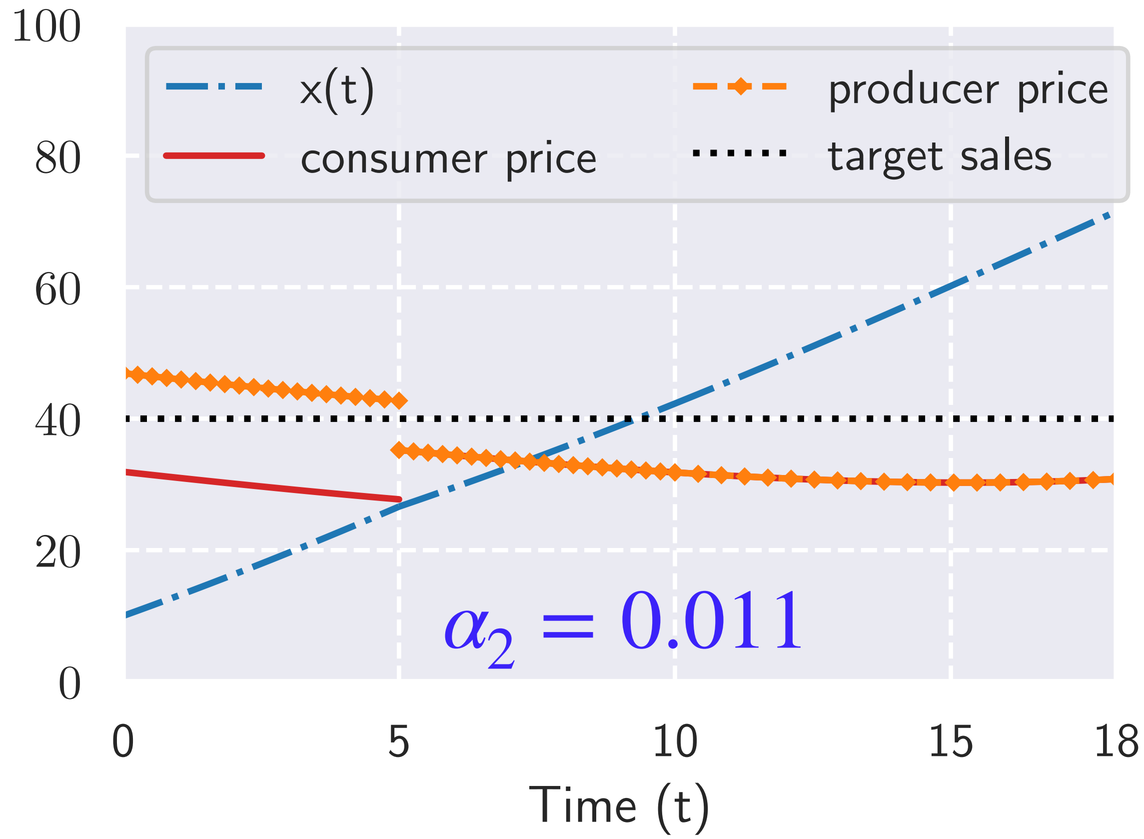


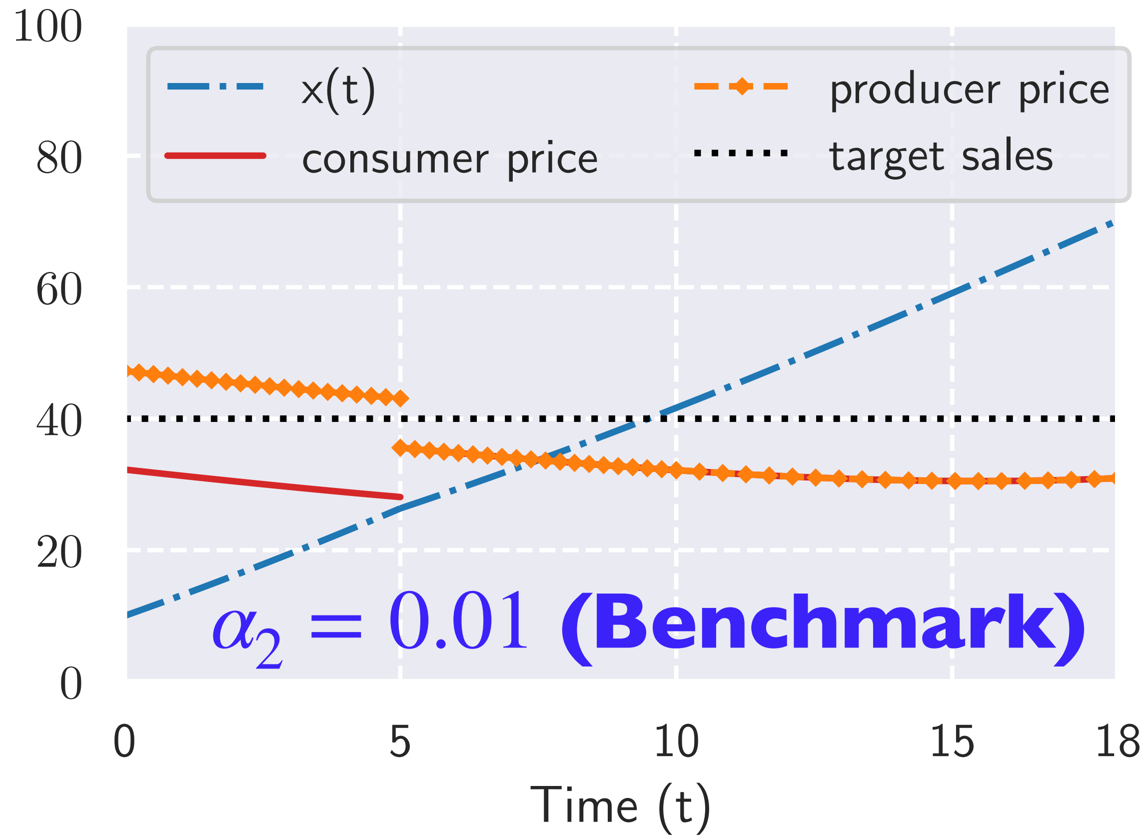
Impact of learning speed



- Lower price and lower subsidy
- Subsidy budget is reduced by almost 3 times

Impact of word of mouth





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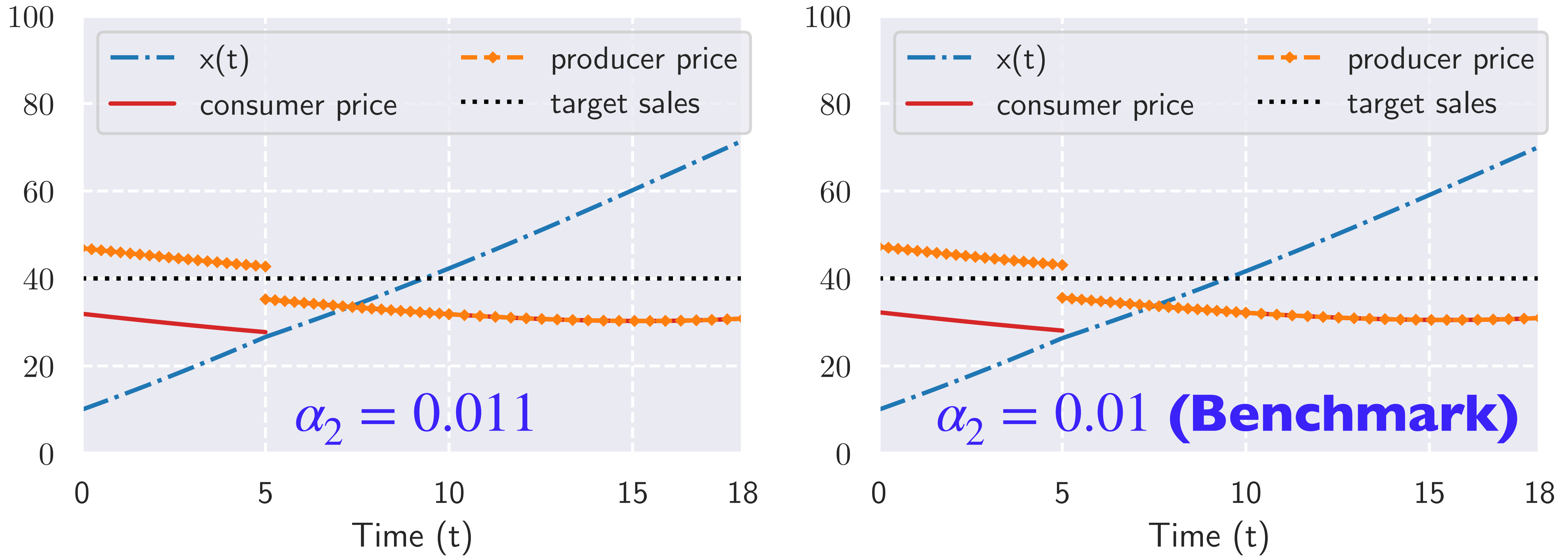
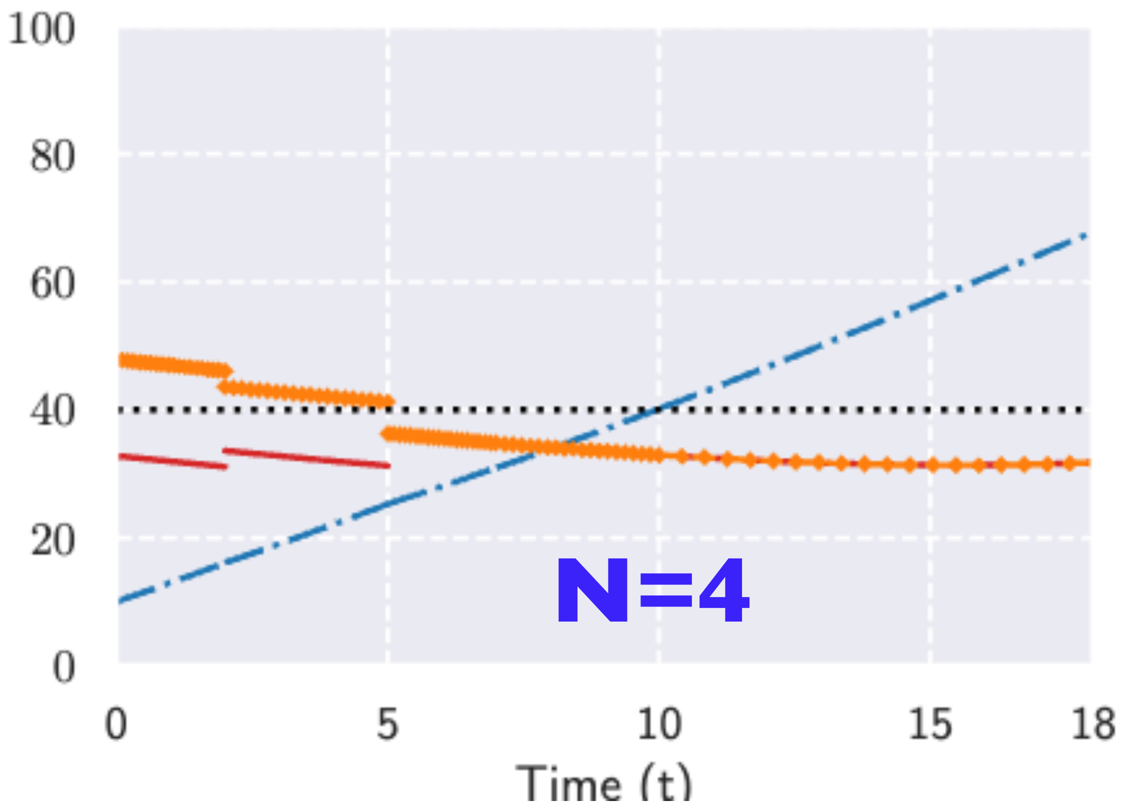
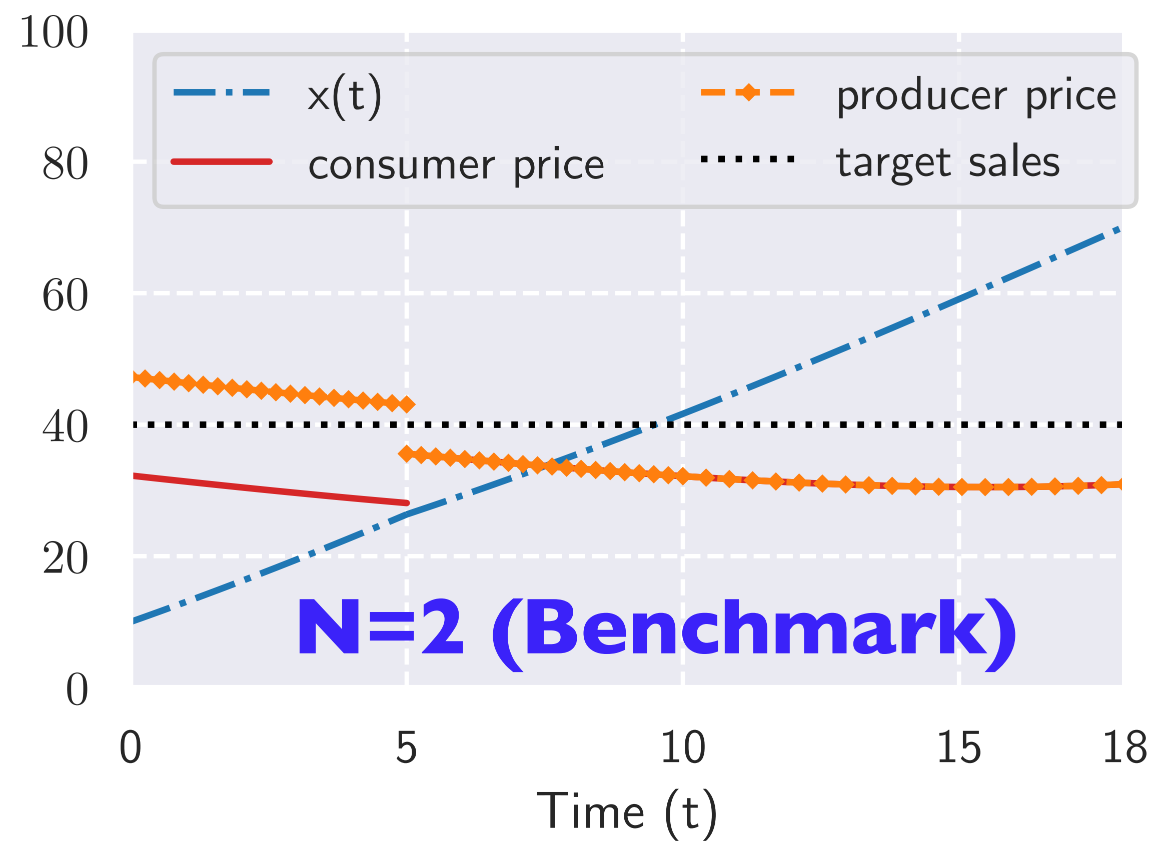
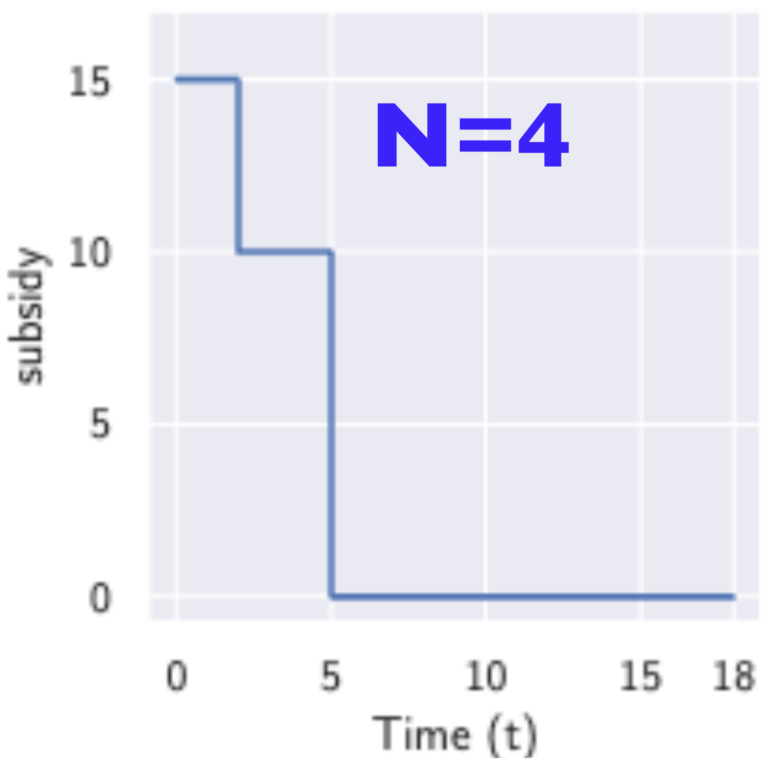


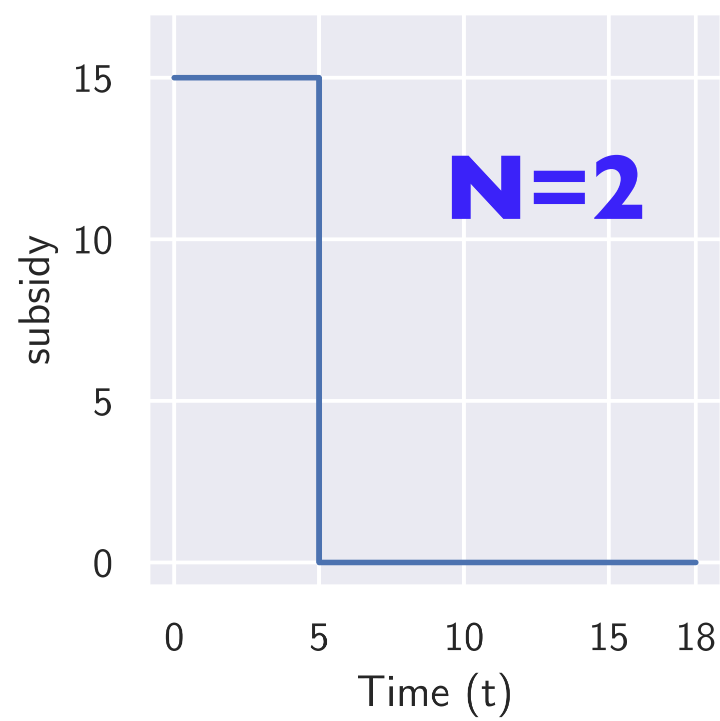
- No significant difference in price, subsidy and adoption of EVs
- High market potential reduces the incentive to lower prices

Impact of number of decision dates









Take-away messages

- Compute feedback equilibria in Stackelberg impulse differential games
- Extensions
  - Hyperbolic discounting (Solve PDE)
  - Real-world case study
  - Stochastic case
  - Timing the interventions
  - Solve Quasivariational inequality

Link to paper

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